Trading and Hedging Bitcoin Volatility

SIAM Seminar in Mathematical Finance

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Section 1

Physical Volatility and Price Discovery

Price indices and other data complexities

Trading behaviour of bitcoin and ether in centralised exchanges.

Alexander C. and M. Dakos (2020) A Critical Investigation of Cryptocurrency Data and Analysis *Quantitative Finance*, 20(2), 173-188

Alexander, C. and D. Heck (2020) Price Discovery in Bitcoin: The Impact of Unregulated Markets. *Journal of Financial Stability* 50, 1-18.

Alexander, C., Choi, J., Massie, H. and S. Sohn (2020) Price Discovery and Microstructure in Ether Spot and Derivatives Markets. *International Review of Financial Analysis*, 71

Alexander C., Choi, J., Park, H., and S. Sohn (2019) BitMEX Bitcoin Derivatives: Price Discovery, Informational Efficiency and Hedging Effectiveness. *Journal of Futures Markets*, 40(1) 23-43

Section 2

Bitcoin Options and Variance Swaps

Bitcoin implied volatility and variance risk premium.

Alexander, C. and A. Imeraj (2020) The Bitcoin VIX and its Variance Risk Premium. *Journal of Alternative Investments* DOI: 10.3905/jai.2020.1.112

Live-streaming a bitcoin implied volatility index (CryptoCompare)

Alexander, C. and A. Imeraj (2020) BVIN: The Bitcoin Implied Volatility Index. CryptoCompare

Section 3

Hedging with Perpetuals

Perpetual futures contracts and default rates

Modelling the impact of margin constraints and default aversion on optimal hedging of bitcoin spot price volatility.

Alexander, C., Deng, J. and Zou, B. (2021) Optimal Hedging with Margin Constraints and Default Aversion – Application to Bitcoin Perpetual Futures. ArXiv 2101.01261 and SSRN 3760048

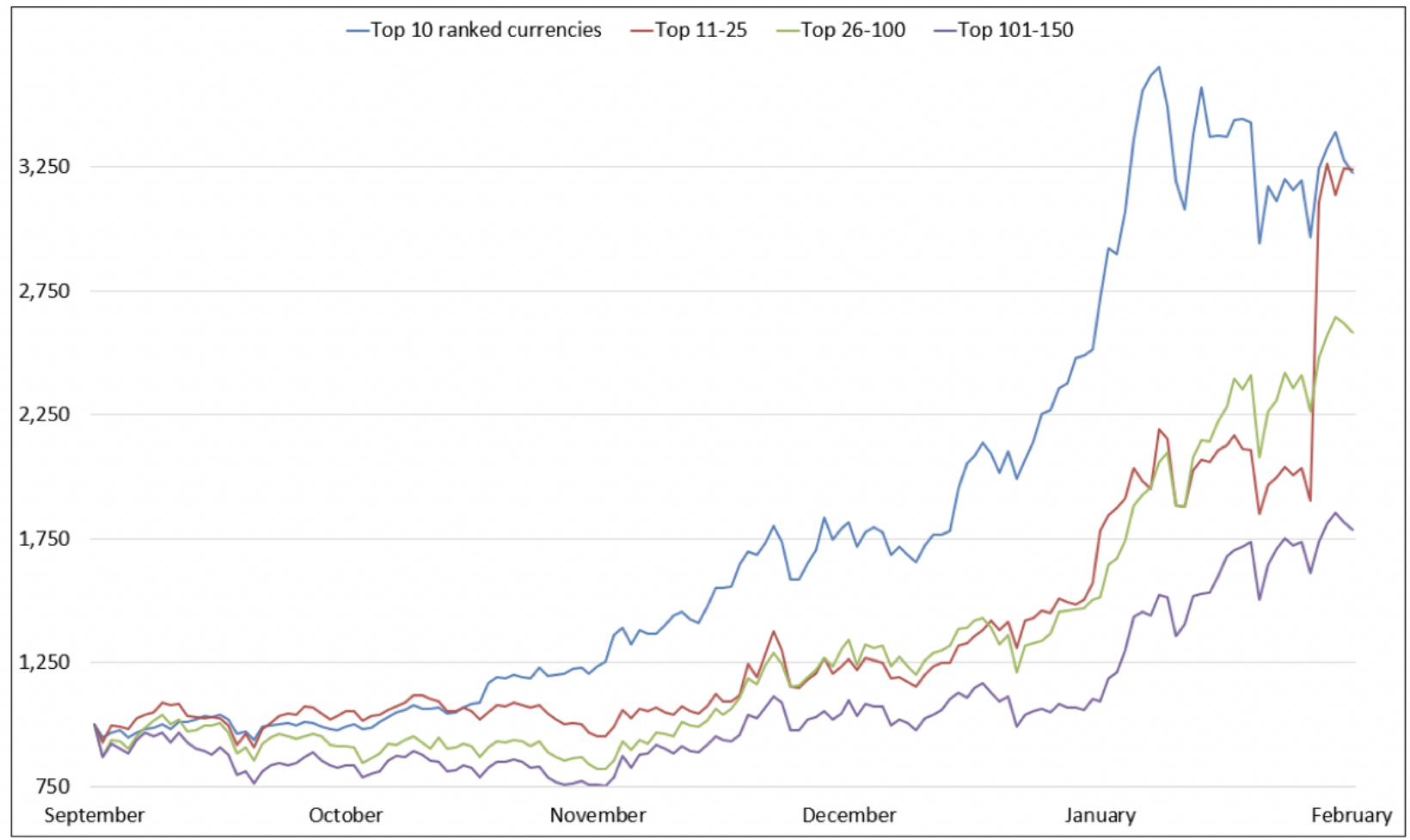
General Overview

Name	Current Price	7d %	30d Vol	90d Vol	21d EMA	50d EMA
Bitcoin	$33,602.69	2.57%	107.41%	82.41%	$33,410.91	$30,474.29
Ethereum	$1,313.77	-7.69%	161.26%	114.96%	$1,248.43	$1,050.49
XRP	$0.548	103.57%	202.26%	220.70%	$0.3148	$0.3260
Polkadot	$15.98	-15.43%	210.99%	160.28%	$15.25	$11.98
Cardano	$0.35	-3.81%	179.70%	141.58%	$0.3283	$0.2742
Chainlink	$22.42	-10.80%	183.93%	136.34%	$20.97	$17.92
Litecoin	$130.18	-9.72%	135.37%	128.32%	$137.60	$128.58
Bitcoin Cash	$398.86	-10.75%	178.69%	137.42%	$431.50	$404.15
Stellar	$0.32	18.54%	257.98%	213.25%	$0.2756	$0.2366

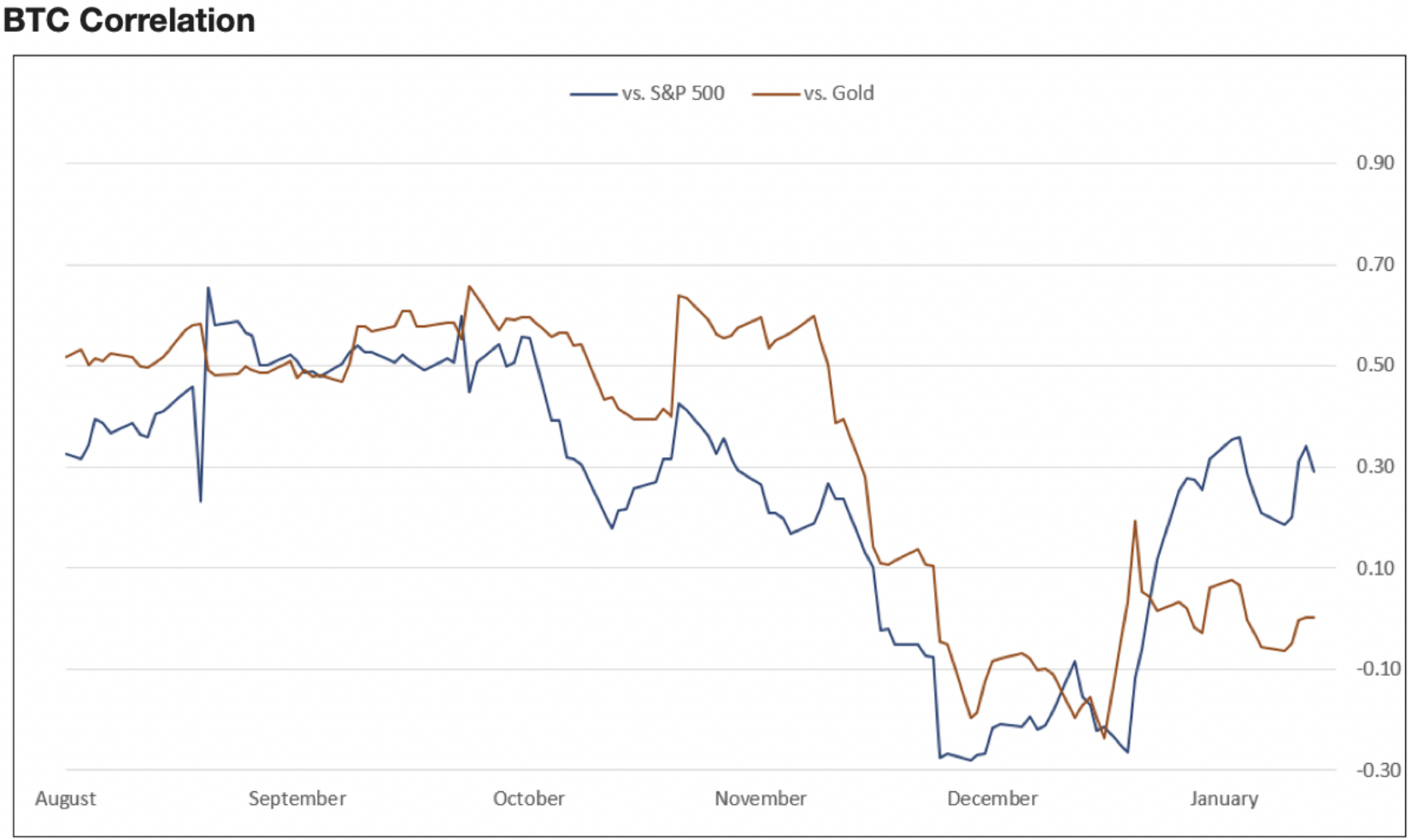
**As of 2/1/2021 4:05 am UTC*

Source: GSG Digital Asset Management

Sector Returns



Correlation of BTC Returns with Gold and US Equities



Main Players in the Crypto Ecosystem

Issuers

Stable Coins

Ranking Sites

CryptoCompare

Exchanges

coinbase

Also central banks, regulators, software and data analytics companies

Ranking Sites

COINS | FORUM | NEWS

Featured — BlockFi - Earn up to 8.6% APY with a BlockFi Interest Account. Get up to $250 deposit bonus in crypto.

#	Coin	Price	Direct Vol	Total Vol	Top Tier Vol	Market Cap		Last 7 Days	24h
1	Bitcoin BTC	$ 33,599.31	$ 1.64 B	$ 11.30 B	$ 11.10 B	$ 625.50 B	B+		0.68%
2	XRP XRP	$ 0.4119	$ 953.33 M	$ 9.42 B	$ 9.22 B	$ 41.19 B	C+		-12.32%
3	Ethereum ETH	$ 1,323.05	$ 834.55 M	$ 6.77 B	$ 6.56 B	$ 151.48 B	A-		-0.68%
4	Dogecoin DOGE	$ 0.03928	$ 212.69 M	$ 3.81 B	$ 3.61 B	$ 5.03 B	C		33.74%
5	Stellar XLM	$ 0.3078	$ 253.45 M	$ 1.67 B	$ 1.57 B	$ 15.39 B	B-		-1.85%
6	EOS EOS	$ 2.989	$ 57.27 M	$ 1.40 B	$ 1.12 B	$ 3.07 B	C		2.43%
7	Litecoin LTC	$ 128.73	$ 86.60 M	$ 1.00 B	$ 926.15 M	$ 8.61 B	B-		-1.03%
8	Uniswap Protocol UNI	$ 18.10	$ 135.22 M	$ 968.83 M	$ 830.23 M	$ 18.10 B			-0.33%
9	Bitcoin Cash BCH	$ 410.96	$ 51.12 M	$ 798.32 M	$ 764.52 M	$ 7.66 B	C+		2.21%
10	Chainlink LINK	$ 21.87	$ 86.82 M	$ 594.50 M	$ 585.57 M	$ 21.87 B			-3.27%

CryptoCompare

Stable Coins

3.70%
SECTOR DOMINANCE

$125.63B
SECTOR 24H VOLUME

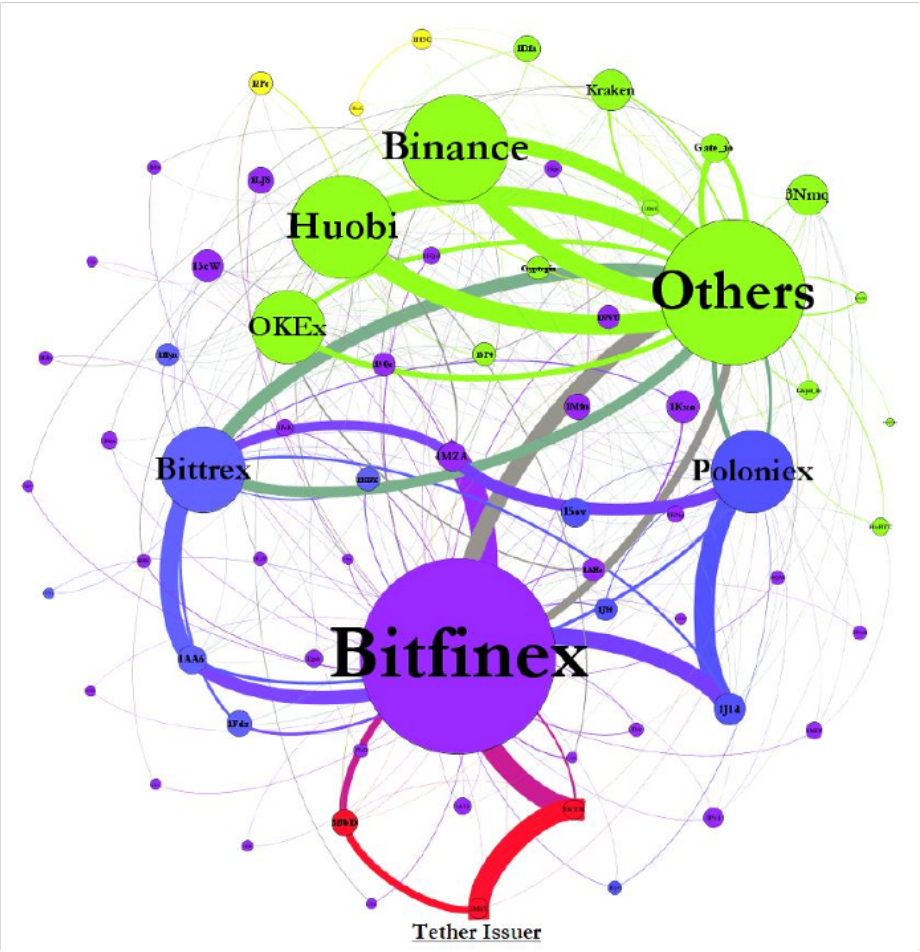
-0.01%
SECTOR 24H CHANGE

Search for a coin...

Showing **1 - 39** of **39** coins

#	Name	Marketcap	Price	24h Volume	Blockchain	Change 24h	Change 7d
1	**Tether** USDT	$26.62B	$1.00067	$121,092,885,343	Omni	+0.01%	-0.01%
2	**USD Coin** USDC	$5.88B	$1.00004	$1,440,879,553	Ethereum	-0.01%	-0.01%
3	**Dai** DAI	$1.66B	$1.00034	$257,723,933	Ethereum	-0.01%	-0.08%
4	**Binance USD** BUSD	$1.5B	$0.99990	$2,443,767,229	Ethereum	-0.01%	+0.01%
5	**HUSD** HUSD	$463.25M	$0.99908	$159,469,292	Ethereum	-0.12%	-0.13%
6	**TrueUSD** TUSD	$404.13M	$1.00025	$103,321,895	Ethereum	+0.04%	-0.03%
7	**TerraUSD** UST	$268.55M	$1.02914	$27,967,577	Ethereum	-1.06%	+2.86%
8	**Paxos Standard** PAX	$245.23M	$1.00112	$264,933,272	Ethereum	+0.32%	-0.06%
9	**JUST** JST	$73.16M	$0.03237	$133,006,196	Tron	+5.49%	+2.12%
10	**xDai** STAKE	$63.05M	$15.6919	$5,090,741	Ethereum	-13.51%	+1.68%

Tether and BitFinex



Griffin and Shams (2020)

New York's investigation of cryptocurrency Tether hits fresh delay

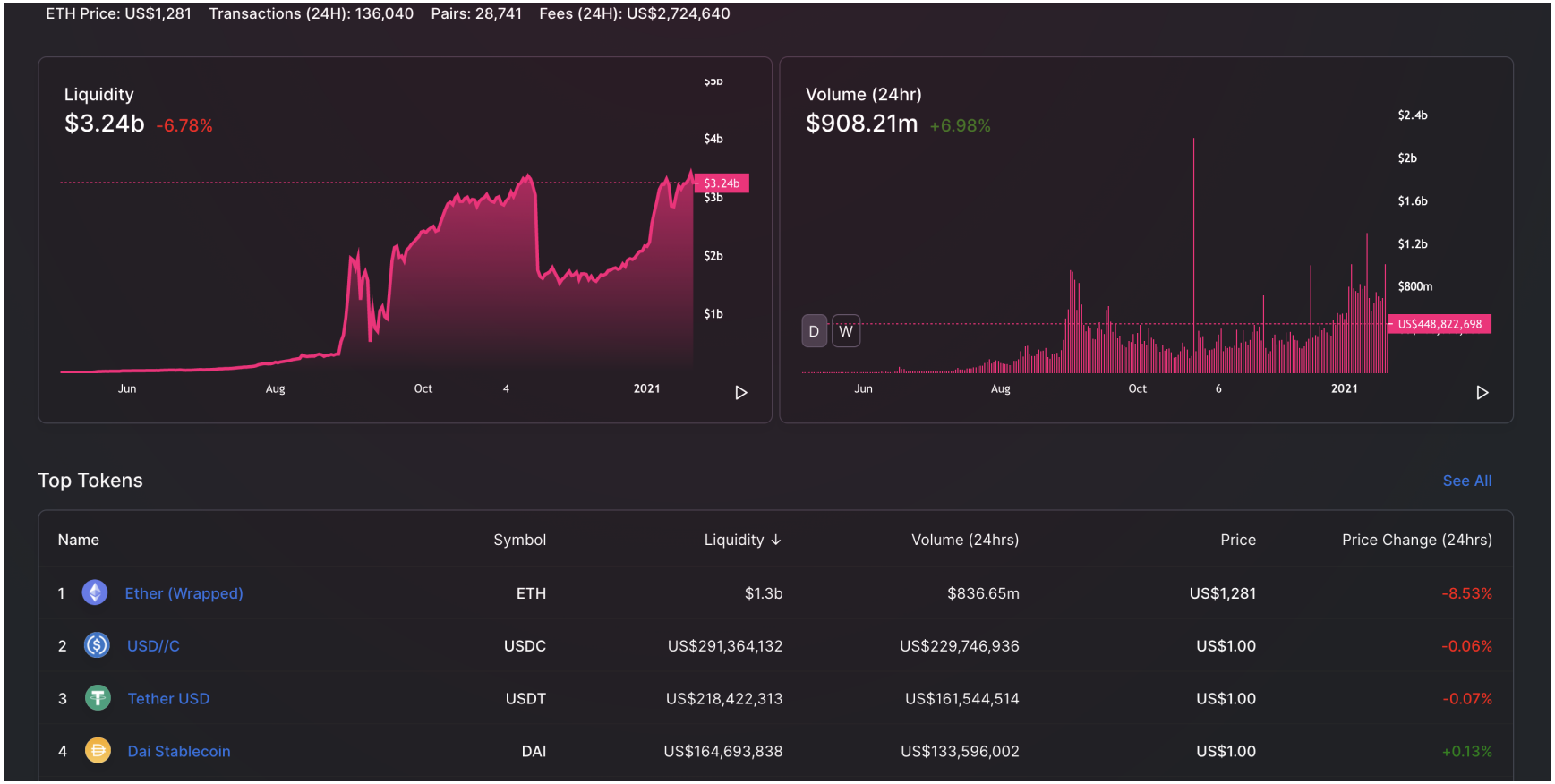
Parent company faces allegation of 'covering up' loss of $850m in company and customer funds

iFinex owns both Tether and trading platform Bitfinex © REUTERS

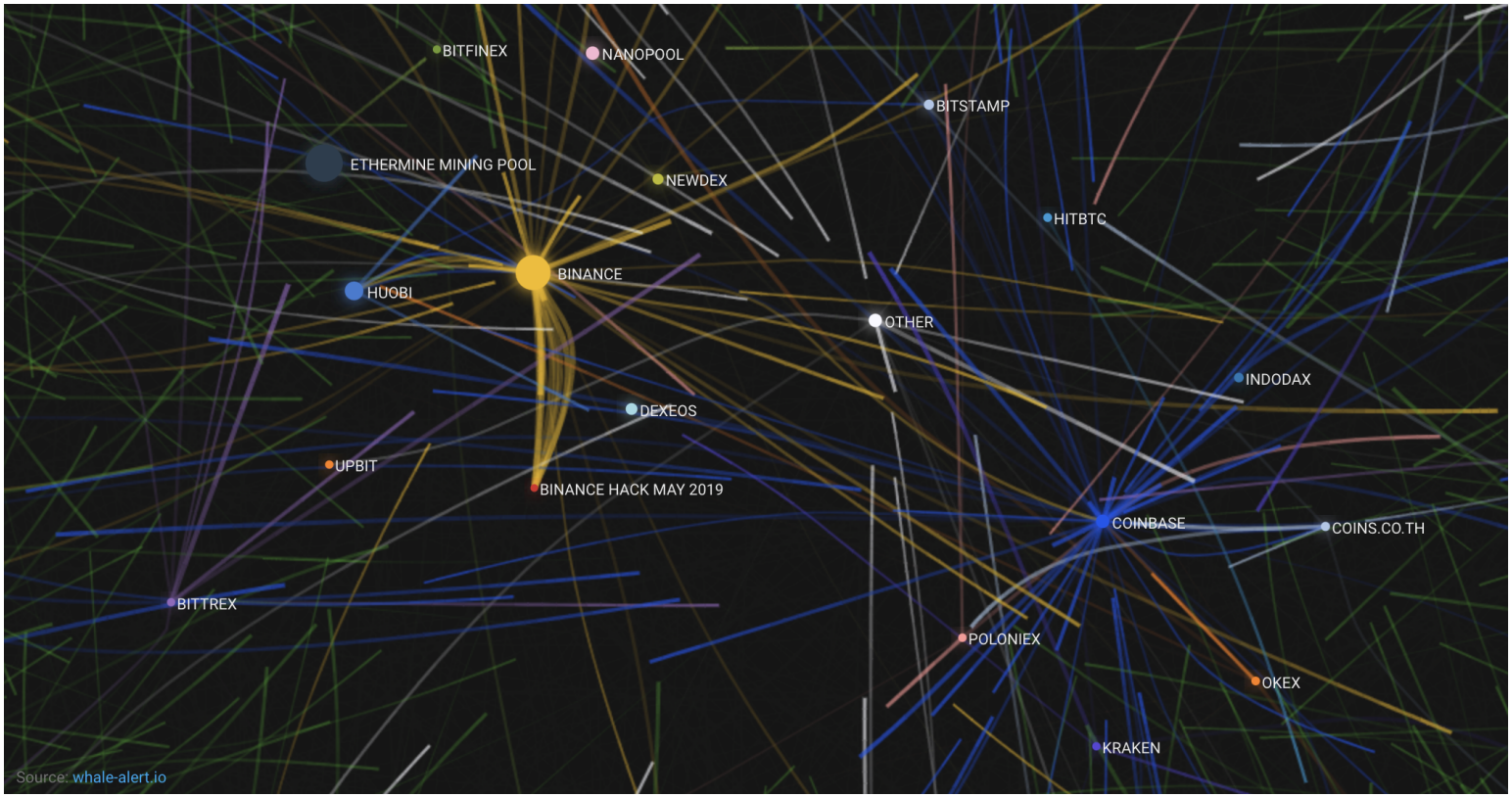
Eva Szalay in London YESTERDAY

New York's investigation into the Tether cryptocurrency and its parent company has suffered a fresh delay, leaving investors uncertain over the outlook for one of the industry's most closely watched cases.

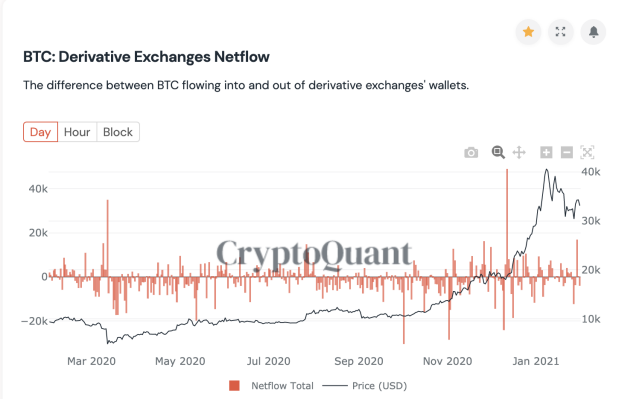
On-Chain Exchanges

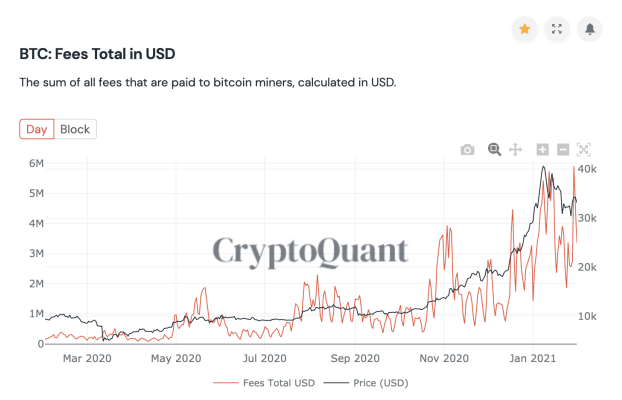


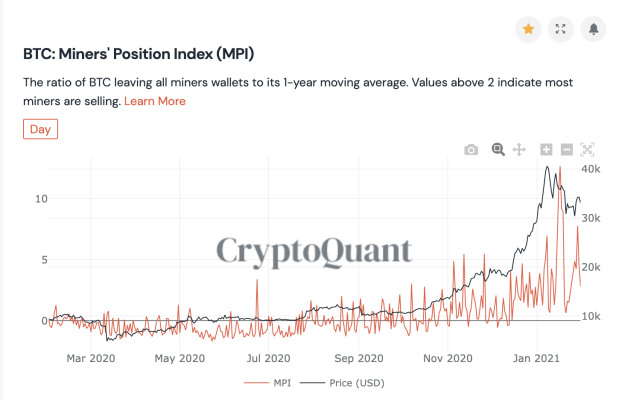
Large on-Chain Flows



On-Chain Data







1: Physical Volatility and Price Discovery

Alexander and Dakos (2020)

Market betas (t-statistics in parentheses) of daily returns on BTC prices from Coingecko (CG), Coinmarketcap (CM) and Cryptocompare (CC), and from Bitfinex, Coinbase, Gemini, Kraken and Poloniex. The market factor is the return on either the CCi30, the CRIX, or the MVDA25 crypto market index. The sample period is 1 April 2016 – 31 March 2019.

	CG	CM	CC	Bitfinex	Coinbase	Gemini	Kraken	Poloniex
CCi30	0.374	0.730	0.744	0.742	0.734	0.743	0.734	0.743
	(14.6)	(44.3)	(44.9)	(43.4)	(43.2)	(44.1)	(43.5)	(43.3)
CRIX	0.903	0.528	0.519	0.515	0.506	0.515	0.497	0.521
	(69.4)	(20.9)	(20.1)	(19.6)	(19.4)	(19.7)	(18.9)	(19.8)
MVDA25	0.350	0.495	0.509	0.501	0.504	0.508	0.504	0.504
	(15.4)	(24.1)	(24.6)	(23.7)	(24.2)	(24.4)	(24.3)	(23.8)

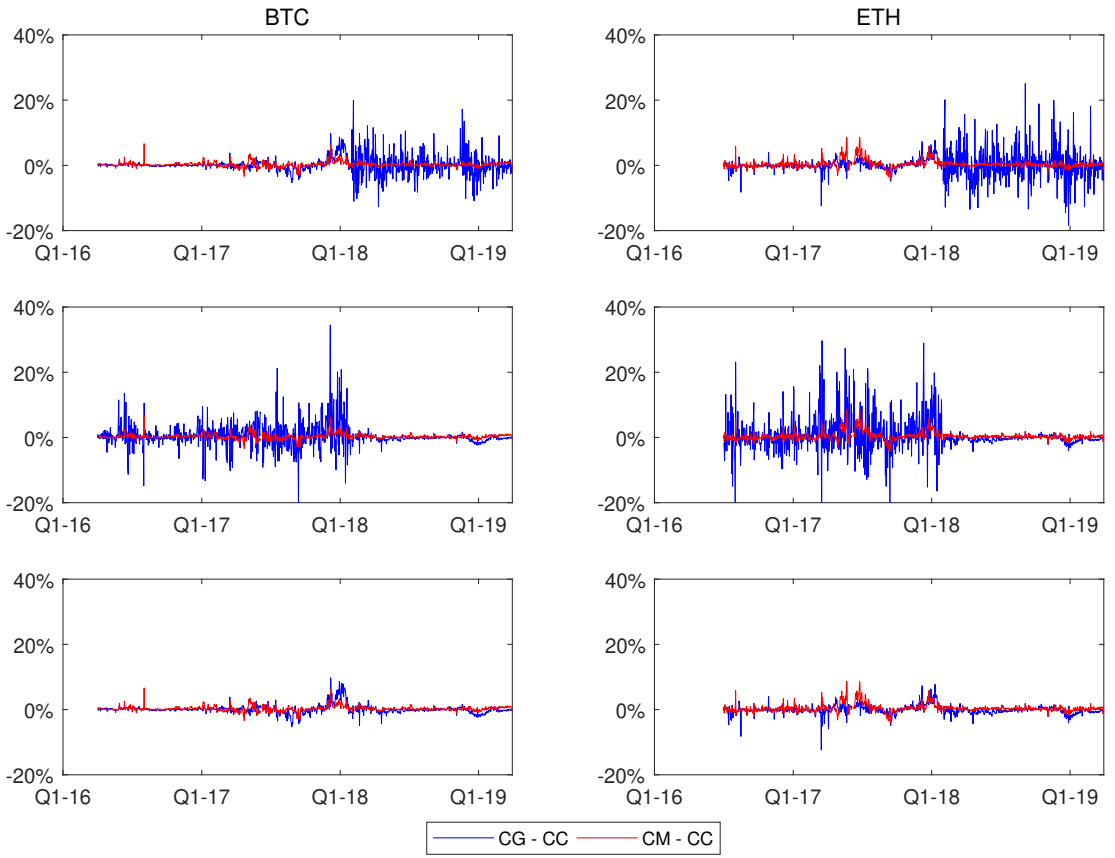
Market betas (t-statistics in parentheses) of daily returns on ETH prices from CG, CM and CC, and from Bitfinex, Coinbase, Gemini, Kraken and Poloniex. The market factor is the return on either the CCi30, the CRIX, or the MVDA25 crypto market index.

1 July 2016 – 31 March 2019.

	CG	CM	CC	Bitfinex	Coinbase	Gemini	Kraken	Poloniex
CCi30	0.501	1.008	1.020	1.012	1.012	1.023	0.998	1.021
	(13.1)	(37.3)	(37.4)	(37.7)	(36.1)	(36.0)	(36.3)	(37.8)
CRIX	1.041	0.513	0.502	0.489	0.490	0.498	0.471	0.510
	(33.6)	(12.0)	(11.6)	(11.4)	(11.2)	(11.2)	(10.9)	(11.8)
MVDA25	0.568	0.763	0.778	0.757	0.778	0.777	0.760	0.772
	(16.8)	(25.3)	(25.6)	(25.0)	(25.4)	(24.8)	(25.0)	(25.5)

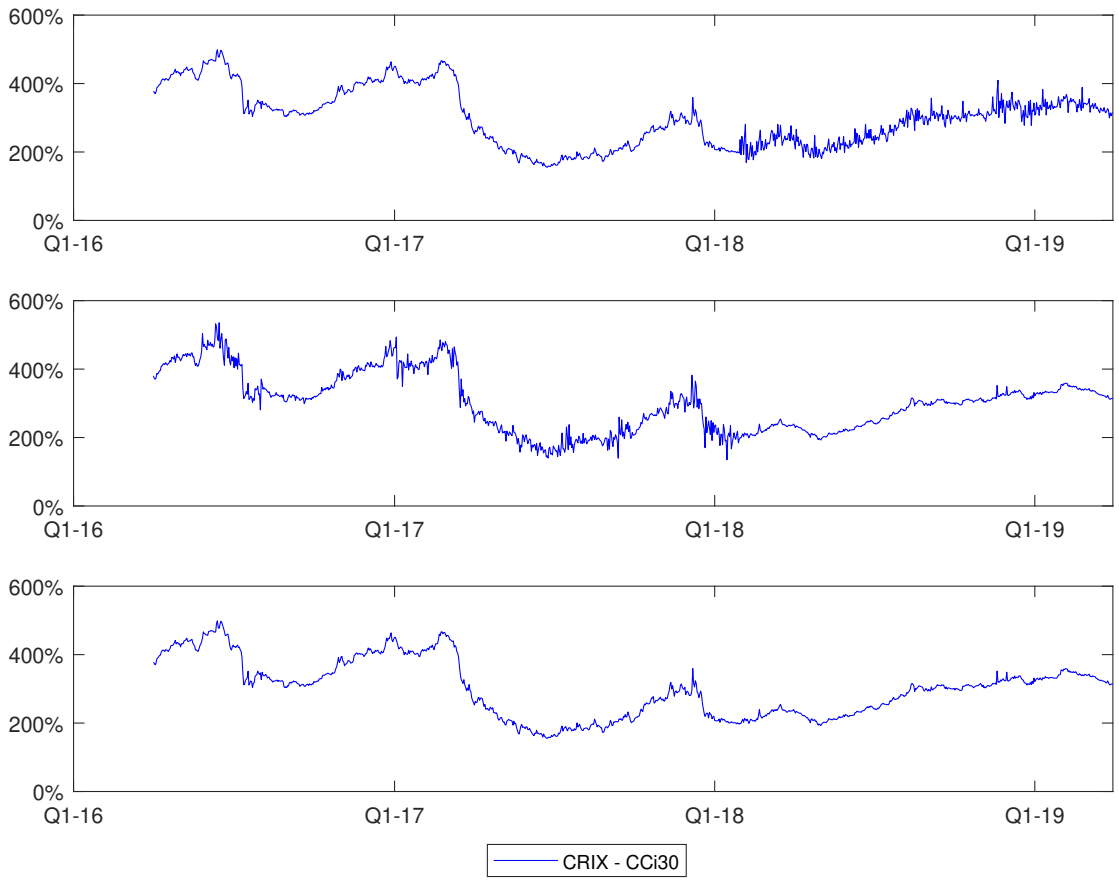
Comparison of VWAP Bitcoin and Ether Rates

CG – CC and CM – CC price spreads relative to CC's daily price for BTC (left-hand graphs) and ETH (right-hand graphs), using CG price data as is (upper), lagged by one day (middle s), and as is until 29 January 2018 and lagged from 30 January 2018 (lower graphs).



Comparison of Crypto Market Indices

CRIX – CCi30 spread relative to CCi30, using CRIX daily data as is (upper graph), lagged by one day (middle graph), and lagged starting from 30 January 2018 (lower graph).



Skew Student t Asymmetric Markov Switching GARCH

Markov switching GJR-GARCH on BTC daily returns from CG, CM and CC and from the Bitstamp and Kraken CEXs. Sample period is 1 September 2013 – 31 March 2019.

	CG	CM	CC	Bitstamp	Kraken
			Low Volatility State		
ω_1	0.145	0.007	0.197	0.036	0.011
α_1	0.147	0.052	0.156	0.070	0.100
γ_1	**-0.067**	**-0.019**	**-0.097**	**-0.006**	**-0.074**
β_1	0.865	0.953	0.858	0.922	0.917
η_1	2.948	2.396	2.873	2.832	3.057
ξ_1	1.013	1.046	0.998	1.028	1.100
p_{11}	0.94	0.96	0.98	0.98	0.86
UV_1	49.57	21.45	45.70	36.32	14.22

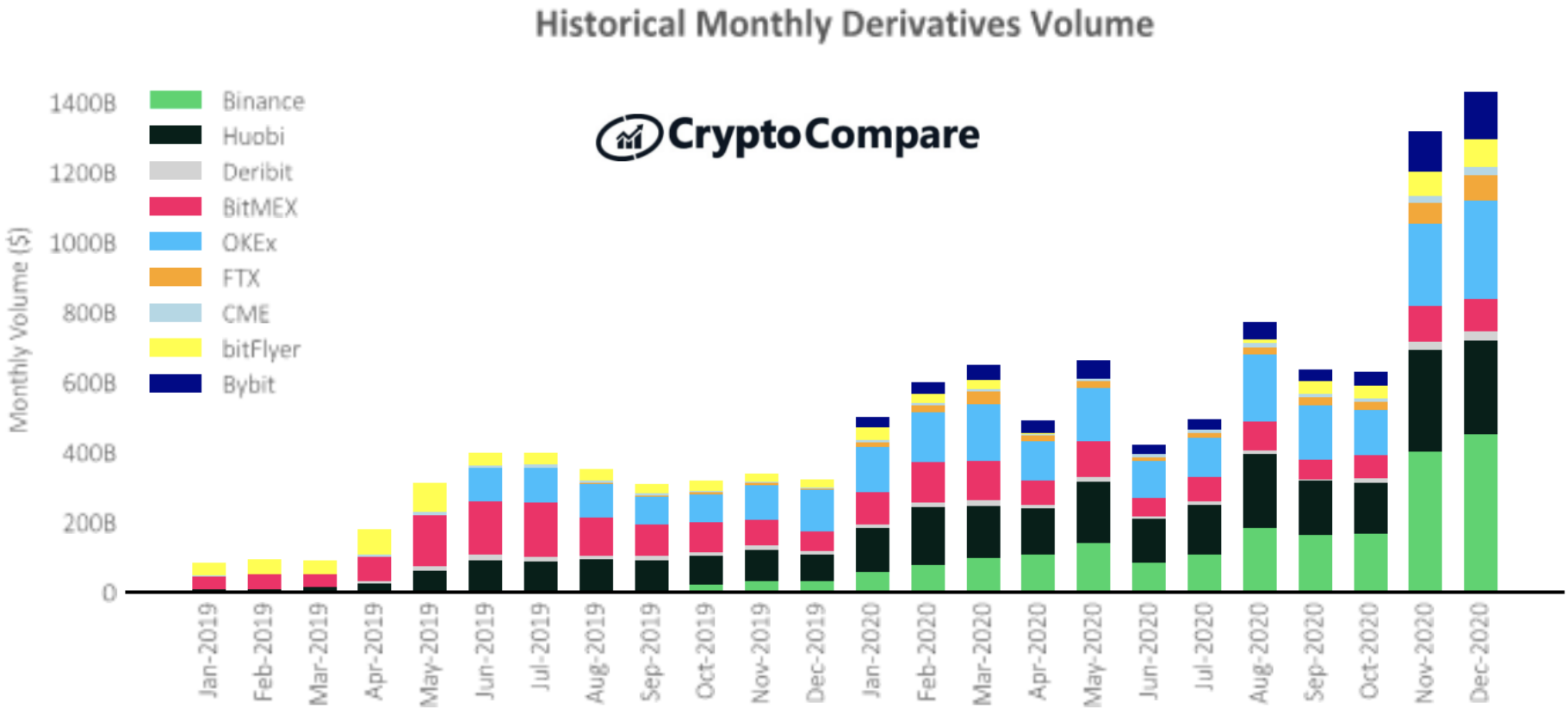
Skew Student t Asymmetric Markov Switching GARCH

Markov switching GJR-GARCH on BTC daily returns from CG, CM and CC and from the Bitstamp and Kraken CEXs. Sample period is 1 September 2013 – 31 March 2019.

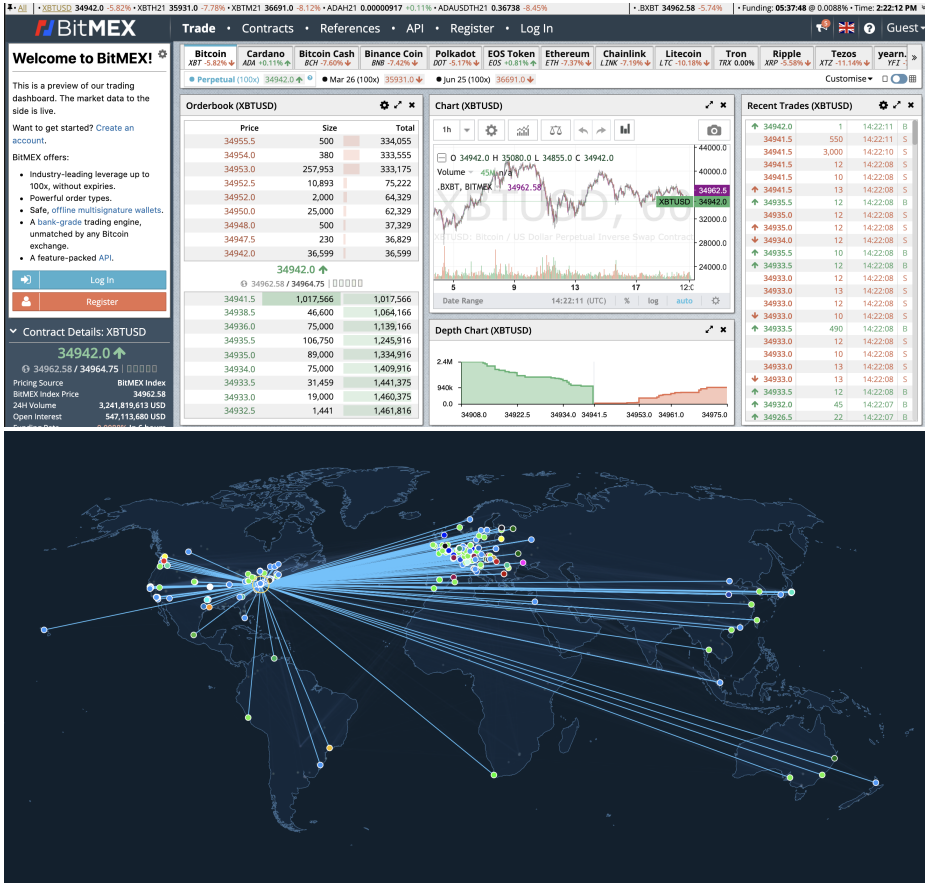
	CG	CM	CC	Bitstamp	Kraken
			High Volatility State		
ω_2	30.442	2.377	4.599	24.312	1.828
α_2	0.003	0.240	0.067	0.231	0.022
γ_2	**0.297**	**-0.055**	**0.204**	**0.311**	**0.168**
β_2	0.009	0.767	0.682	0.308	0.880
η_2	51.933	4.172	4.165	3.131	65.485
ξ_2	0.710	0.865	0.937	0.854	0.906
p_{22}	0.77	0.95	0.97	0.95	0.60
UV_2	115.07	204.41	106.27	170.28	216.24

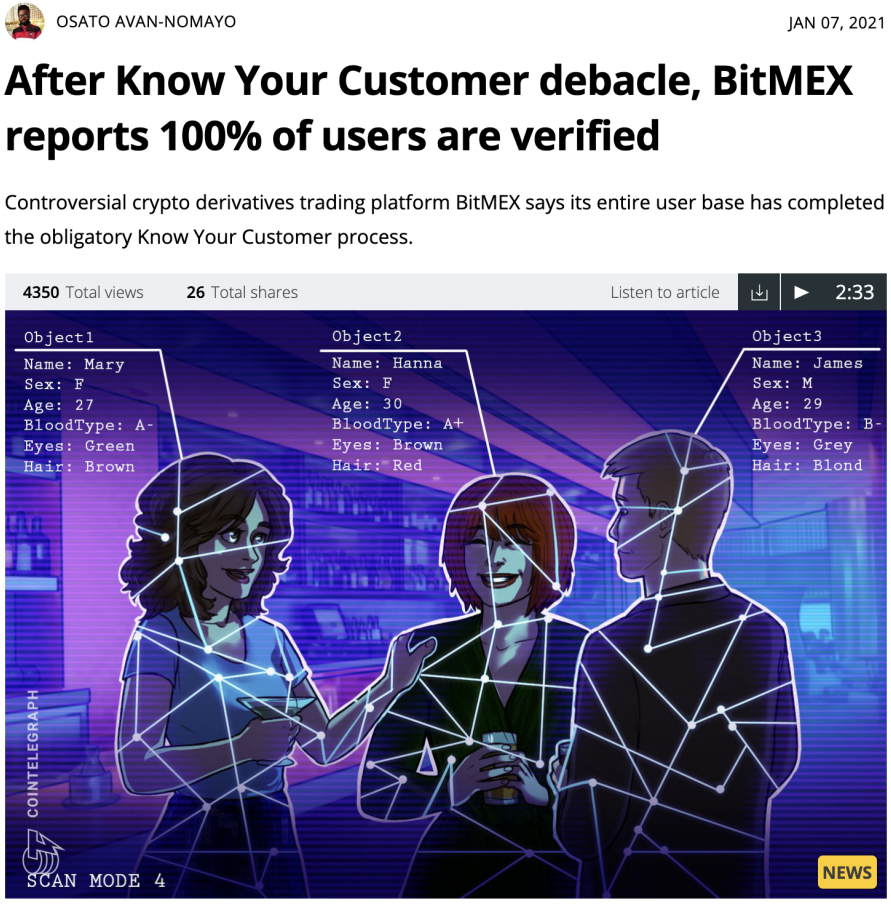
Growth in Futures Trading Volumes

Derivatives volumes increased 8.6% in December to an all-time monthly high of $1.43tn. Meanwhile, total spot volumes increased by 30% to $1.19tn. The derivatives market now represents almost 54.6% of the total crypto market (vs 60% in November).



BitMEX Exchange





BitMEX

Binance Exchange

Dogecoin Perpetual

Alexander and Heck (2020)

Let $\mathbf{p}_t$ be the $n \times 1$ vector of cointegrated log prices at time t and let $z_t = \boldsymbol{\beta}^T \mathbf{p}_t$ denote their deviations from long-run equilibrium. Then the VECM is:

$$\Delta \mathbf{p}_t = \boldsymbol{\alpha} + \sum_{i=1}^{q-1} \boldsymbol{\Gamma}_i \Delta \mathbf{p}_{t-i} + \boldsymbol{\delta} z_{t-1} + \mathbf{e}_t,$$

where $\mathbf{e}_t$ are serially uncorrelated innovations with zero mean and covariance matrix $\boldsymbol{\Omega}$ and $\boldsymbol{\delta}$ captures reactions to transitory equilibrium deviations. Inverting and integrating gives:

$$\mathbf{p}_t = \mathbf{p}_0 + \boldsymbol{\Psi}(1) \sum_{j=1}^{t} \mathbf{e}_j + \boldsymbol{\Psi}^\star(L)\mathbf{e}_t$$

where $\boldsymbol{\Psi}(1)$ i.e. the sum of the MA coefficients in the inversion of the AR, has identical rows which we denote $\boldsymbol{\psi}$. Then the scalar $\boldsymbol{\psi}\mathbf{e}_t$ is the common efficient price and it has variance $\boldsymbol{\psi}\boldsymbol{\Omega}\boldsymbol{\psi}^T$

Price Discovery Methodology

The Hasbrouck (1995) information share of each market, i.e. its relative contribution to the variance of the common efficient price, is:

$$\text{IS}_i = \frac{([\boldsymbol{\psi}\mathbf{M}]_i)^2}{\boldsymbol{\psi}\boldsymbol{\Omega}\boldsymbol{\psi}^T} \quad \text{for } i = 1, \ldots, N,$$

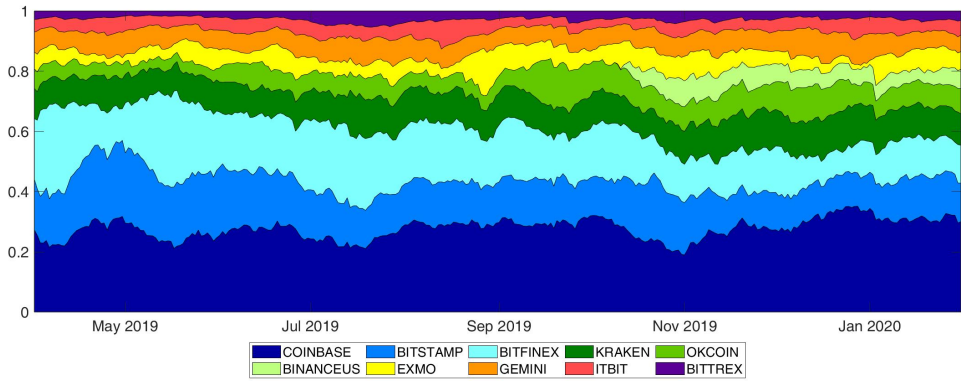
where $\mathbf{M}$ is the lower triangular matrix of the Cholesky decomposition of $\boldsymbol{\Omega}$ and $[\boldsymbol{\psi}\mathbf{M}]_i$ is the i-th entry of $\boldsymbol{\psi}\mathbf{M}$.

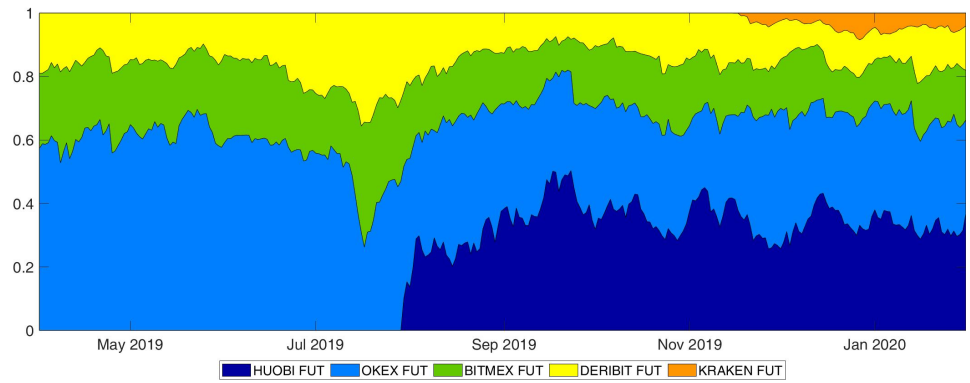
Various improvements of the IS have been proposed for different reasons – we use the generalised information share of Lien and Shrerestha (2015)

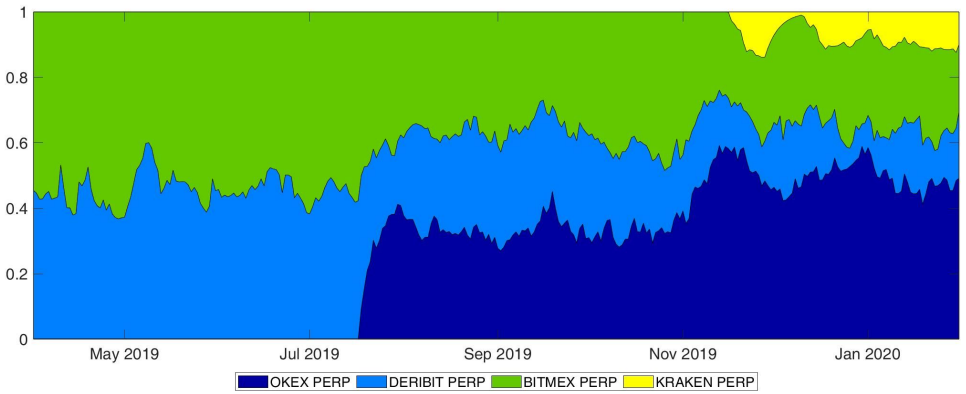
We also check robustness with the component share of Gonzalo and Granger (1995)

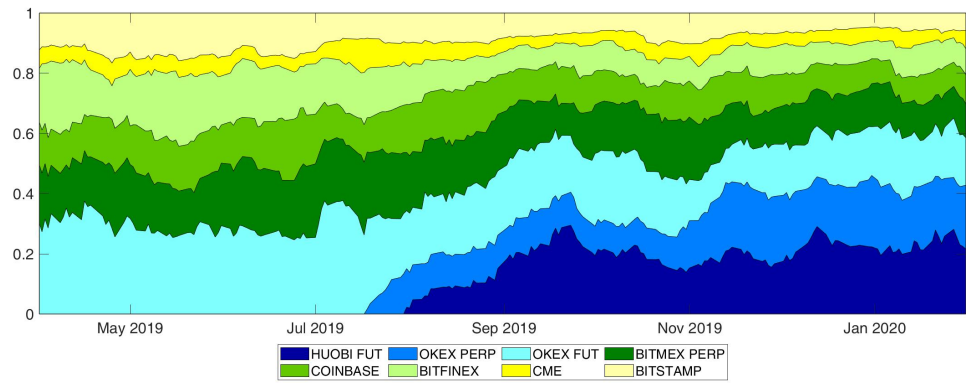
Results

	Regulated futures		Unregulated futures				
Contracts	CME	Bakkt	BitMEX	Deribit	Huobi	Kraken	OKEx
Contract size	5 XBT	1 XBT	1 USD	10 USD	100 USD	1 USD	100 USD
Frequency	Monthly	Daily, monthly	Quarterly	Quarterly	Weekly, Quarterly	Monthly, Quarterly	Weekly, quarterly
Margin requirement	37%	37%	1%	1%	1%	2%-6%*	10%
Settlement	Cashin USD	Physical	Cashin XBT	Cashin XBT	Cash in XBT	Cash in XBT	Cash in XBT
Trading days	Weekdays	Weekdays	24/7	24/7	24/7	24/7	24/7
Delivery date	Last friday	3rd friday	Last friday	Last friday	Last friday	Last friday	Last friday
Fees (maker/taker)	$1.25	$1.25	−2.5/7.5	−2/5	2/3**	−2/7.5	2/5**









2. Options and Variance Swaps

ADRIAN ZMUDZINSKI APR 25, 2019

Algorithmic Crypto Trading Firm GSR Launches Crypto Hedging Product

Hong Kong-based algorithmic crypto trading firm GSR launched cryptocurrency variance swaps, a product for hedging against volatility.

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Hong Kong-based algorithmic crypto trading firm GSR launched cryptocurrency variance swaps, a product for hedging against volatility. The company announced the development in a press release published on April 24.

Alexander and Imeraj (2020)

Implied variance from traded options:

$$V_T = T^{-1}\left[2\sum_i K_i^{-2}\mathcal{Q}(K_i,T)\Delta K_i - \left(\frac{F_T}{K_0}-1\right)^2\right]$$

Interpolation to constant maturity gives fair-value variance swap rate:

$$\text{VXBT}_T = \sqrt{\omega T_1 V_{T_1} + (1-\omega)T_2 V_{T_2}}$$

with

$$\omega = \frac{n_2 - n}{n_2 - n_1}N$$

$\mathcal{Q}(K_i,T)$ is the price of OTM option with maturity T and strike K_i
F_T is the price of the futures contract with maturity T
K_0 is separation strike, i.e. puts for strikes $K_i < K_0$ and calls for strikes $K_i > K_0$
N is the number of seconds in a year
n_i is the number of seconds until maturity T_i, for $i = 1, 2$
n is the number of seconds until maturity T

Example – Deribit Option Prices in April 2020

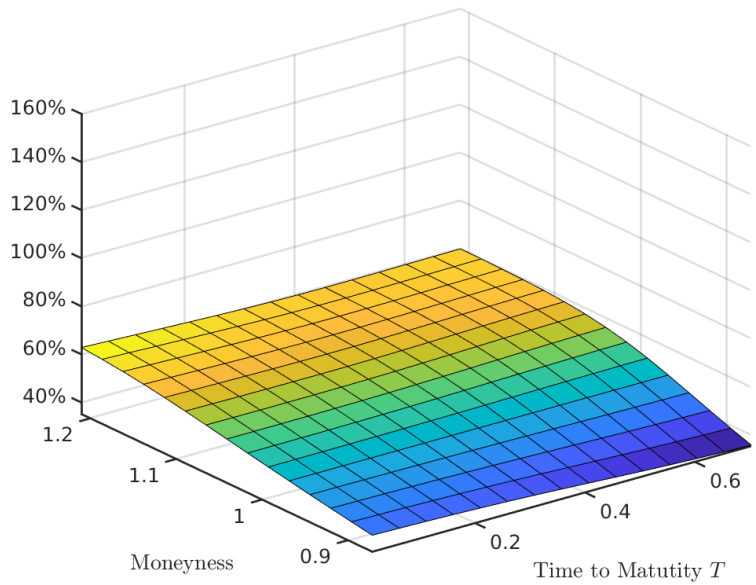
T_1-Strike	Type	Price in $
4250	P	8.967
4375	P	10.25
4500	P	20.5
4625	P	32.03
4750	P	49.97
4875	P	76.86
5000	P	115.31
5125	C	180.66
5250	C	130.69
5375	C	93.53
5500	C	69.19
5625	C	44.84
5750	C	30.75
5875	C	23.06
6000	C	15.375
6125	C	10.25
6250	C	8.97
6375	C	7.69
6500	C	6.41
6625	C	6.41

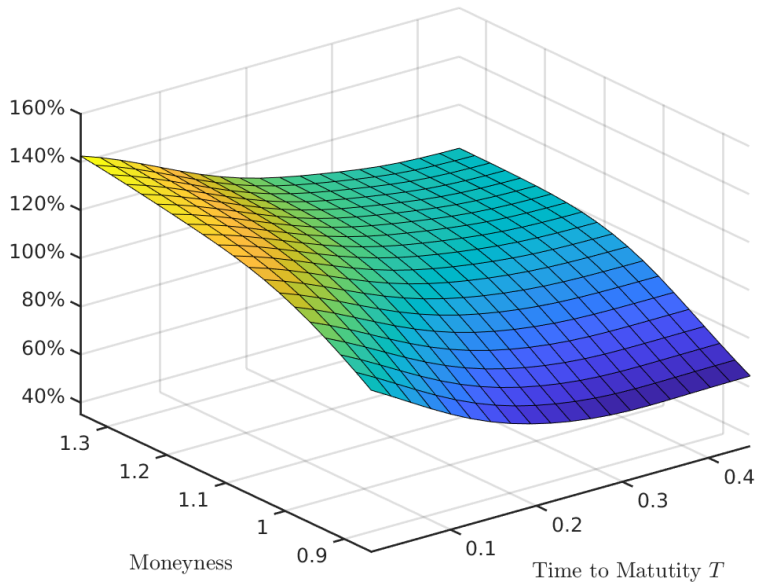
T_2-Strike	Type	Price in $
4000	P	16.66
4125	P	21.78
4250	P	33.31
4375	P	48.69
4500	P	67.91
4625	P	94.81
4750	P	126.84
4875	P	169.13
5000	P	220.375
5125	P	285.72
5250	C	240.86
5375	C	193.47
5500	C	156.31
5625	C	124.28
5750	C	98.66
5875	C	78.16
6000	C	58.94
6125	C	48.68
6250	C	38.44
6375	C	30.75
6500	C	23.06
6625	C	20.5
6750	C	16.66

Deribit

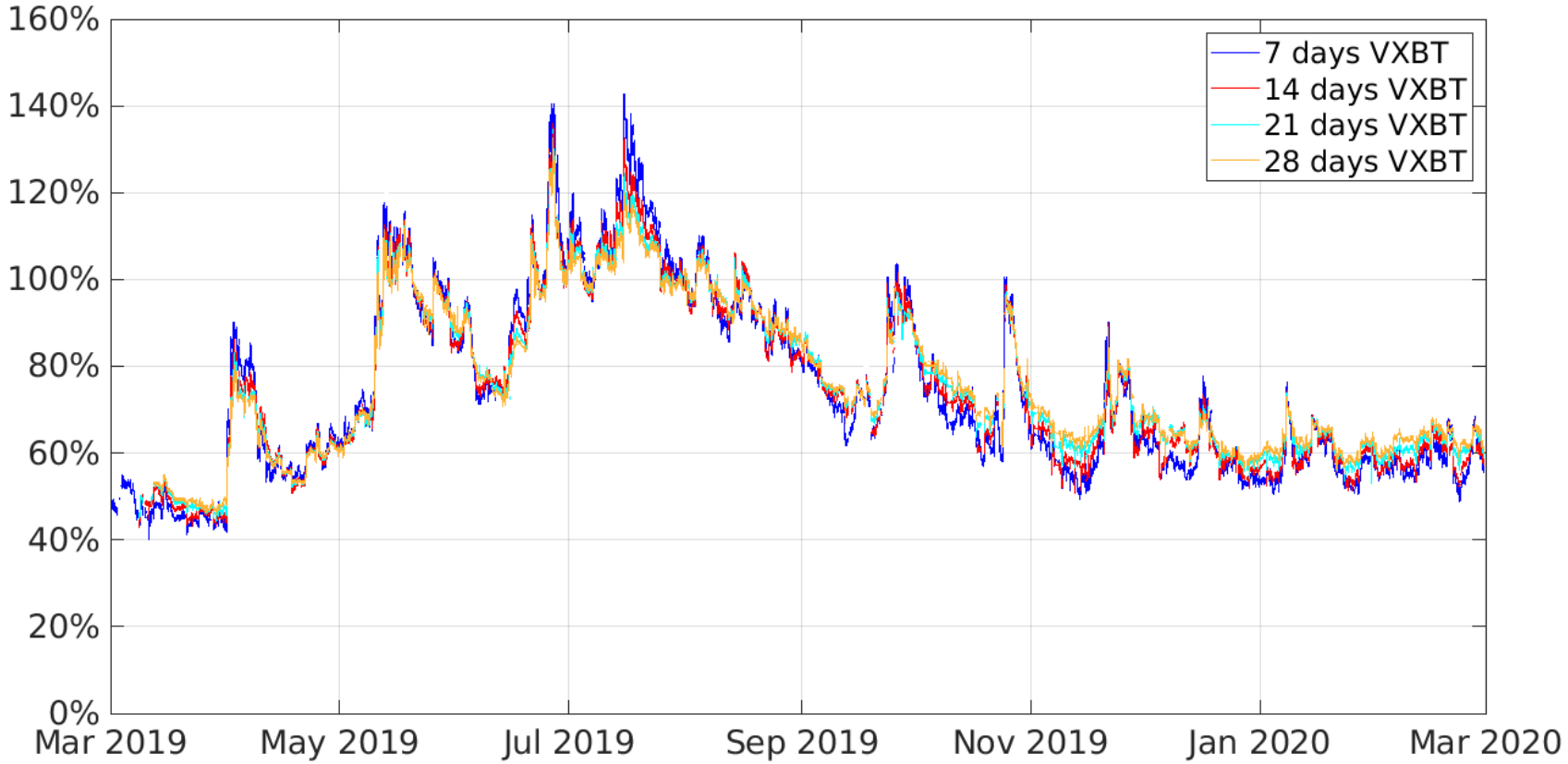
Implied Volatility Indices

Implied volatilities from out-of-the-money options of different strikes and maturities on 15 April 2019 at 02:00:00 UTC (left) and 16 June at 13:00:00 UTC (right). Surfaces interpolated using shape-preserving splines.

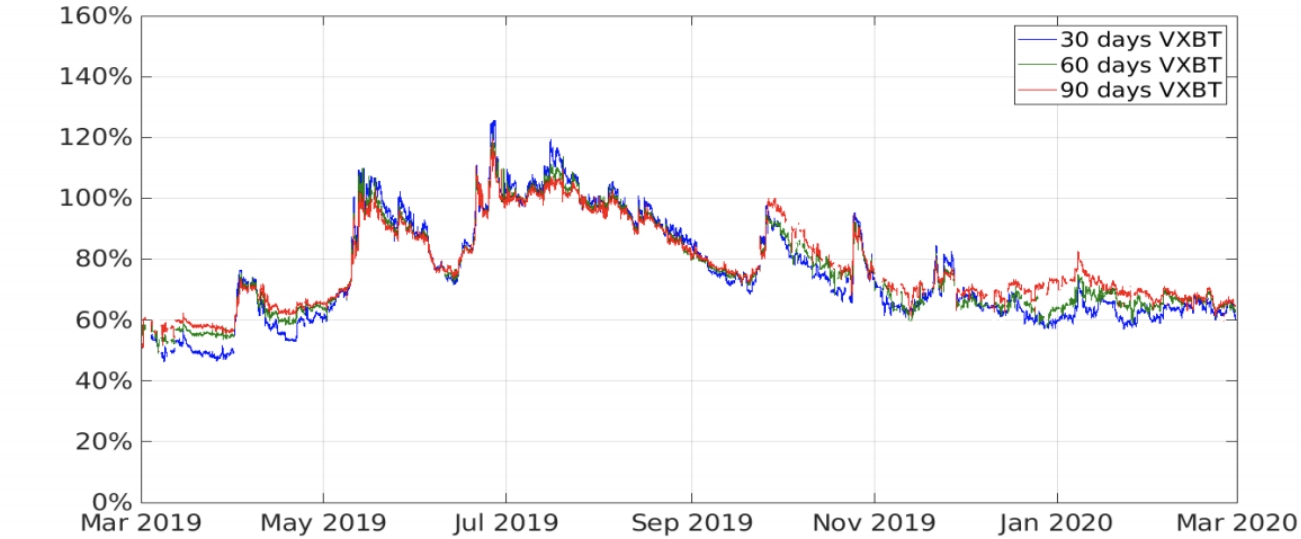


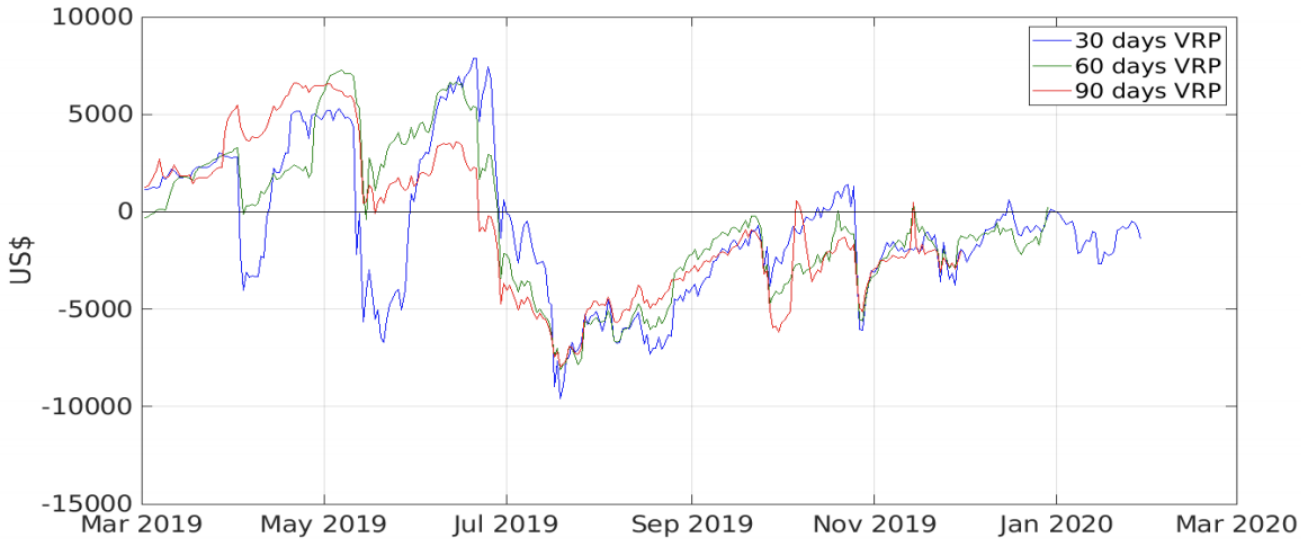


Variance Swap Rate Term Structure (Short End)



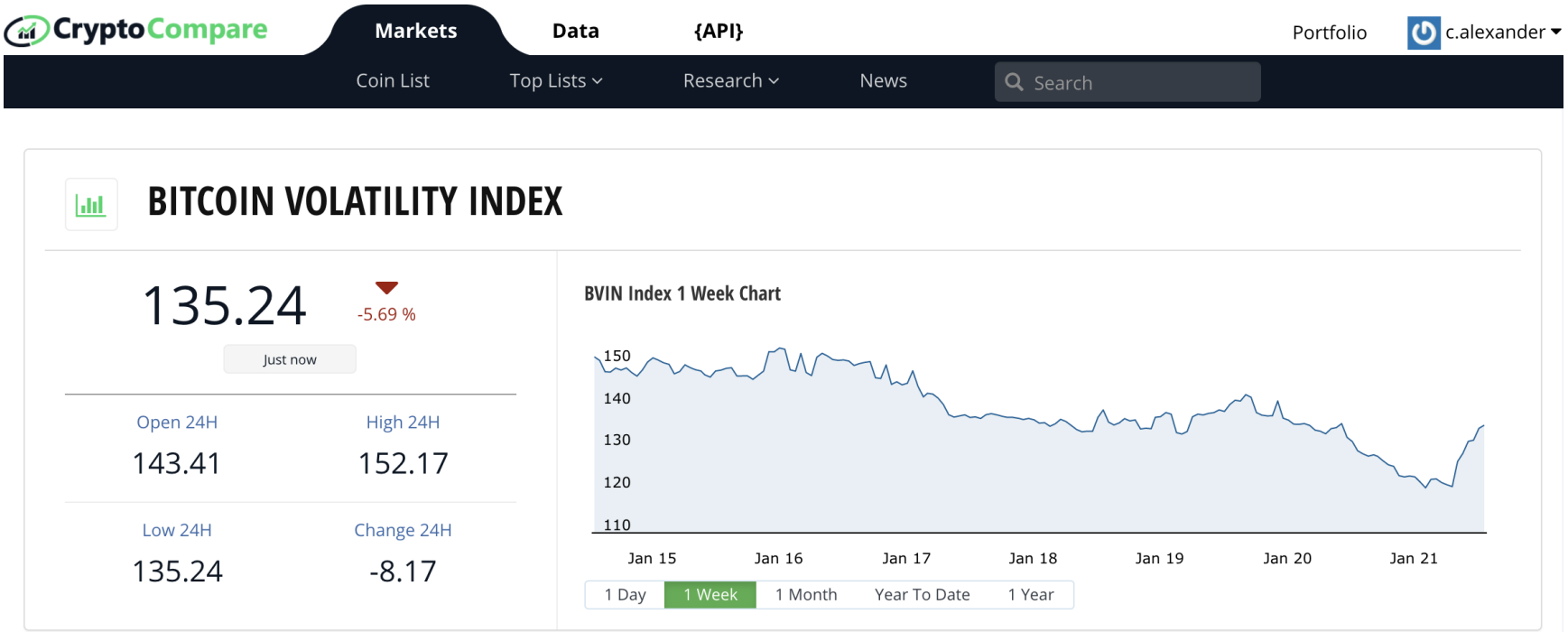
Bitcoin Variance Risk Premium per $1 Notional





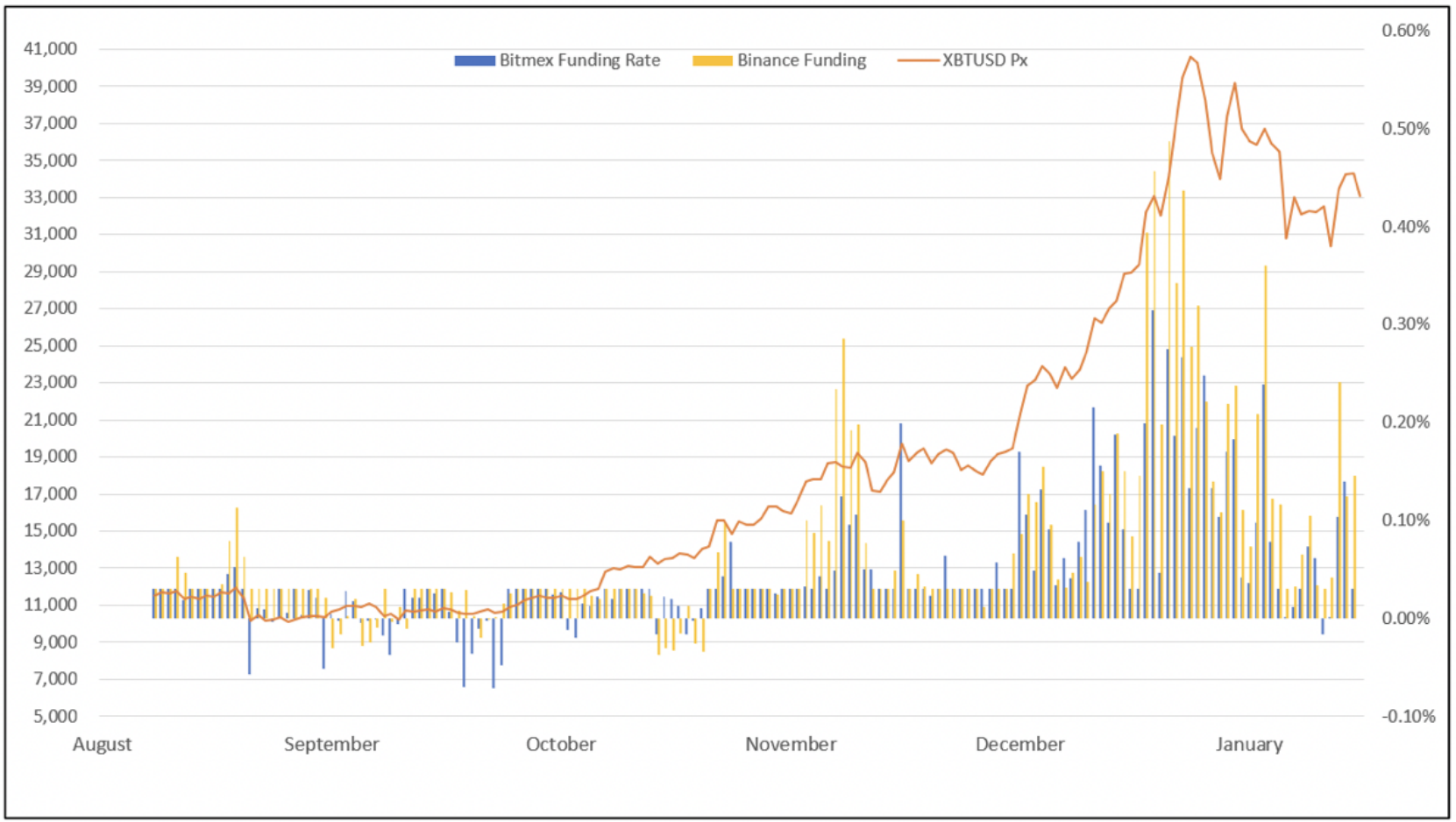
Reverse Leverage Effect in Bubble Regime

CryptoCompare Live-Streamed BVIN Index



BVIN

3. Hedging with Perpetuals



Alexander, Deng and Jun (2020)

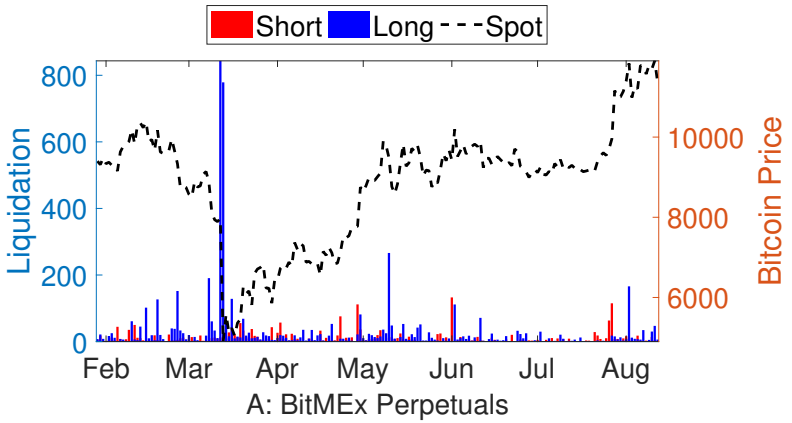
- Perpetuals have no expiry date
- Spot and futures converge via funding rate mechanism
- A positive (negative) rate $\Rightarrow$ P2P payment long $\rightarrow$ ($\leftarrow$) short
- Fluctuates $+$ve/$-$ve in contango/backwardation of futures curve
- e.g. funding rate of -0.05% implies payment of \$50 on \$100,000 notional short $\rightarrow$ long
- Perpetuals are ideal hedging instruments for miners
- But default risk cannot be ignored due to ultra-high leverage

Margin Mechanism

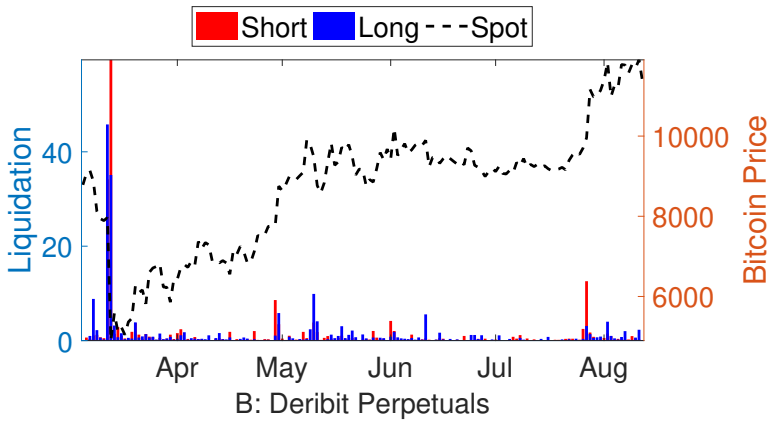
Settlement	Settlements take place every day at 8:00 UTC. Realized and unrealized session profits (profits made between settlements) are always added in real-time to the equity. However, they are only available for withdrawal after the daily settlement. At the settlement, session profits/losses will be booked to the BTC cash balance.
Contract Size	10 USD
Initial Margin	The initial margin starts with 1.0% (100x leverage trading) and linearly increases by 0.5% per 100 BTC increase in the position size. Initial margin = 1% + (Position Size in BTC) * 0.005%
Maintenance Margin	The maintenance margin starts with 0.525% and linearly increases by 0.5% per 100 BTC increase in the position size. When the account's margin balance is lower than the maintenance margin, positions in the account will be incrementally reduced in order to keep the maintenance margin lower than the equity in the account. Maintenance margin requirements can be changed without prior notice if market circumstances demand such action. Maintenance Margin= 0.525% + (Position Size in BTC) * 0.005%
Mark Price	The mark price is the price at which the perpetual contract will be valued during the trading hours. This can (temporarily) vary from the actual perpetual market price in order to protect market participants against manipulative trading. Mark Price = Index price + 30 seconds EMA of (Perpetual Market Price - Index Price) Where market price is the last traded futures price if it falls between the current best bid and best ask. Otherwise, the market price will be the best bid. If the last traded price is lower than the best bid, or market price will be the best ask, if the last traded price is higher than the best ask.
Delivery/Expiration	No Delivery / Expiration

- Leverage on perpetuals is exceedingly high – around 100X
- If trading losses exceed the maintenance margin, most exchanges employ a platform that automatically and incrementally liquidates the position to cover the margin call

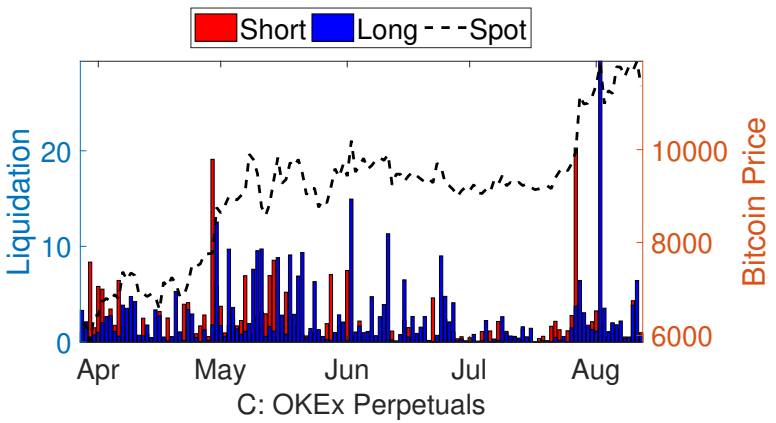
Forced Liquidations on Three Perpetuals ($m)



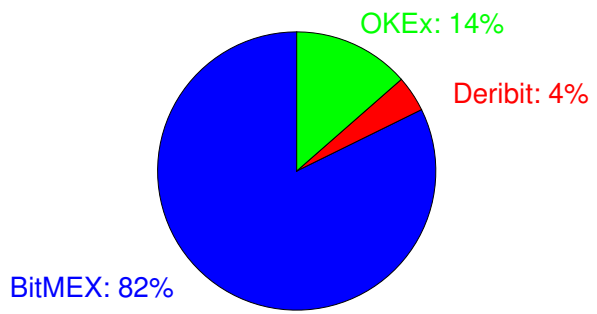
A: BitMEx Perpetuals



B: Deribit Perpetuals



C: OKEx Perpetuals



D: Relative Weight of Liquidations

$800 million worth of long positions in BitMEX perpetuals were forced into liquidation on 12 March 2020. Data from coinanalyse.net

Probability of BitMEX Margin Call

Table 1: Historical Probability of Margin Call for BitMEX Perpetual Futures

	Leverage	5X	15X	20X	50X	100X
Short	1d	0.30%	8.97%	16.05%	44.33%	63.19%
	5d	3.42%	37.26%	47.15%	75.67%	84.79%
	15d	19.54%	65.52%	68.97%	85.06%	91.95%
	30d	41.86%	67.44%	72.09%	83.72%	93.02%
Long	1d	1.44%	12.70%	18.40%	43.73%	61.22%
	5d	9.51%	40.68%	49.81%	67.68%	80.23%
	15d	24.14%	56.32%	66.67%	80.46%	89.66%
	30d	37.21%	76.74%	86.05%	93.02%	95.35%

Note. Here we define a margin call as an event when losses from trading futures exceed the margin deposit over the hedge horizon. We use the 5min historical data of bitcoin perpetual futures on BitMEX from January 1, 2017 to August 12, 2020 to compute the probability of margin calls. We perform computations under different leverage levels, ranging from 5X to 100X, and different hedge horizons, ranging from 1d (day) to 30d.

Basic Problem Formulation

Suppose the platform monitors the margin at discrete intervals Δt

The hedger holds 1 bitcoin at time t and shorts θ units of the futures to hedge spot price volatility until hedge horizon $t + N\Delta t$

Suppose the direct and inverse perpetual futures prices are F_t and $\widehat{F}_t = F_t^{-1}$

Set $\Delta_n S_t := S_{t+n\Delta t} - S_t$ (in USD) and $\Delta_n \widehat{F}_t = \widehat{F}_t - \widehat{F}_{t+n\Delta t}$ (in BTC)

Why is $\Delta_n \widehat{F}_t$ [start − end] price? So that a short inverse perpetual position profits from a decrease in its USD value

Need to multiply $-\theta\, \Delta_n \widehat{F}_t$ by $S_{t+n\Delta t}$ to convert into USD

Realised P&L on the hedged portfolio is $\Delta_N S_t - \theta\, \Delta_N \widehat{F}_t \cdot S_{t+N\Delta t}$

Basic hedging problem is to choose θ to minimize:

$$\sigma^2_{\Delta h}(\theta) := \mathbb{V}\mathrm{ar}\big(\Delta_N S_t - \theta\, \Delta_N \widehat{F}_t\, S_{t+N\Delta t}\big)$$

Margins and Liquidations

Hedger deposits b bitcoin in margin account $(0 < b < 1) \Rightarrow$ leverage is b^{-1}

So $b =$ initial margin requirement (e.g. 1%) plus buffer

Margin account marked each Δt so that at time $t + n\Delta t$:

- $\Delta_n \widehat{F} < 0 \Rightarrow$ marked gain $\theta \Delta_n \widehat{F}_t$ from t to $t + n\Delta t$
- $\Delta_n \widehat{F} > 0 \Rightarrow$ marked loss of $\theta \Delta_n \widehat{F}_t$ from t to $t + n\Delta t$

$\theta \Delta_n \widehat{F}_t$ is cumulative unrealised gain or loss (settlement is daily)

Upper constraint $m = b - M$ on hedgers ability to meet margin calls

M is maintenance margin

Forced liquidation occurs if $\theta \Delta_n \widehat{F}_t > m$ for some n, with $0 < n \leq N$

$\Rightarrow$ Hedger wants to minimize:

$$P(m, \theta) = \mathbb{P}\text{rob}(\text{default event}) := \mathbb{P}\text{rob}\left(\theta \max_{0 \leq n \leq N} \Delta_n \widehat{F}_t > m\right)$$

Hedging Problem and Solution

Introduce a default aversion factor γ to obtain final objective:

$$\min_{\theta>0} \quad \left\{\sigma^2_{\Delta h}(\theta) + \gamma\, \sigma^2_{\Delta S}\, P(m,\theta)\right\}$$

Solution: $$\theta^* = w^{-1}\theta_0$$

where $w = S\widehat{F}^2$ at the time of the hedge and θ_0 is positive real root of

$$a(x)x^{-2} + x - b = 0$$

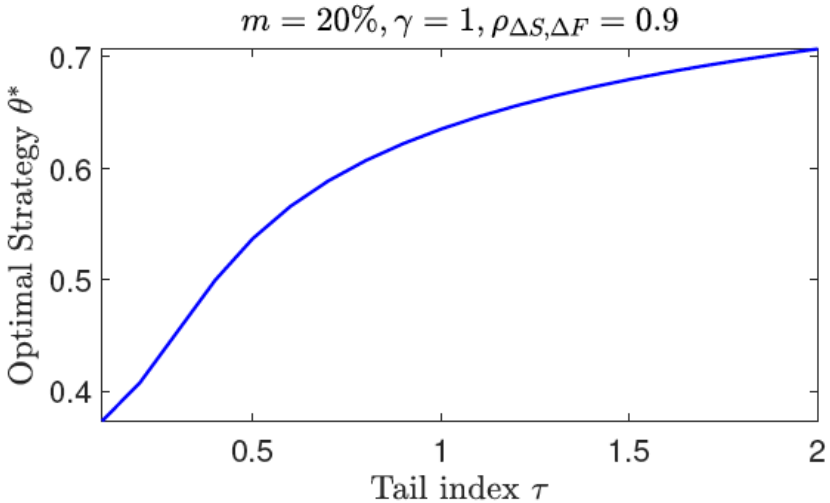
with:

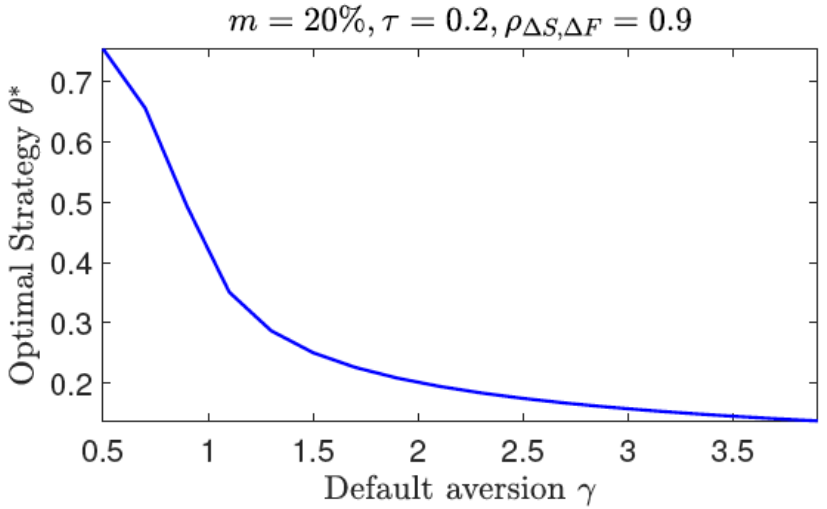
$$a(x) = \frac{\gamma m v^2 w}{2\alpha}\,\frac{\sigma^2_{\Delta S}}{\sigma^2_{\Delta F}}\,\exp\left[-\left(1+\tau\frac{mwx^{-1}-\beta}{\alpha}\right)^{-\frac{1}{\tau}}\right]\left(1+\tau\frac{mwx^{-1}-\beta}{\alpha}\right)^{-\frac{1}{\tau}-1}.$$

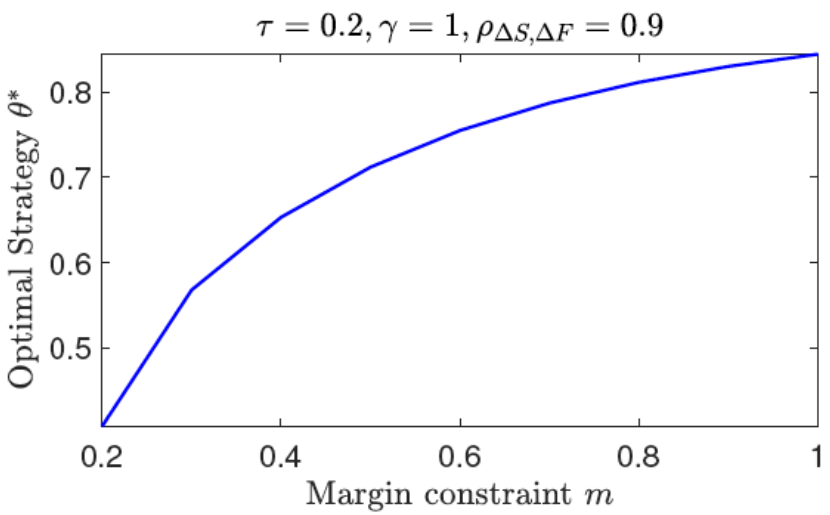
Here $v = \sigma_{\Delta S}/\sigma_{\Delta F}$ is the ratio of n-period spot P&L volatility to that of the direct futures $F_t = \widehat{F}_t^{-1}$ and $b = \frac{\sigma_{\Delta S \Delta F}}{\sigma^2_{\Delta F}}$ where the numerator is the covariance.

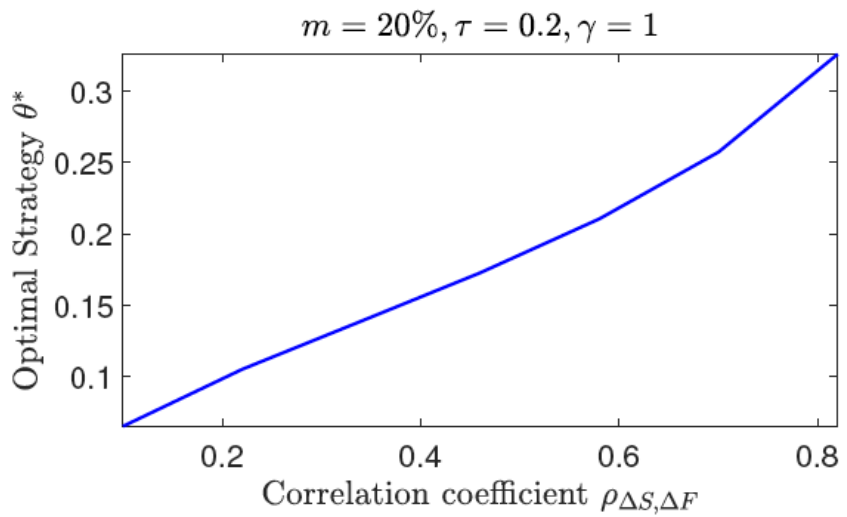
The constants α, β and τ are scale, location and right tail index parameter of the distribution of extreme values of $\Delta\widehat{F}$.

Sensitivity Analysis of Optimal Strategy









Data and Correlations

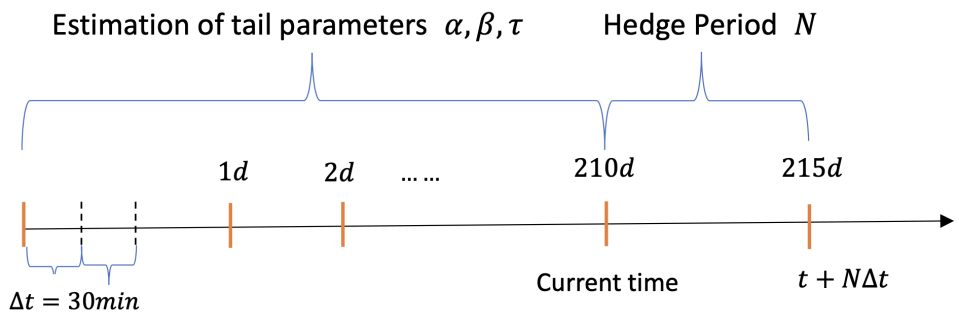
Traded price and volume data are retrieved at the 5-min frequency using coin API. Common available time span is 18 July 2019 to 27 July 2020, about 110,000 observations on each series. The reference price for the BitMEX perpetual futures is calculated at the minute level as the weighted average of the last price of Bitstamp (25.73%), Bittrex (2.63%), Coinbase (46.98%), Gemini (6.69%), and Kraken (17.97%).

		Spot			Perpetuals		
		BitMEX	Bitstamp	Coinbase	BitMEX	Deribit	OKEx
Spot	BitMEX	1.00	0.80	0.81	0.79	0.80	0.79
	Bitstamp	0.80	1.00	0.82	0.87	0.89	0.93
	Coinbase	0.81	0.82	1.00	0.88	0.92	0.96
Perpetuals	BitMEX	0.79	0.87	0.88	1.00	0.92	0.95
	Deribit	0.80	0.89	0.92	0.92	1.00	0.88
	OKEx	0.79	0.93	0.96	0.95	0.88	1.00

Estimation of Tail Parameters

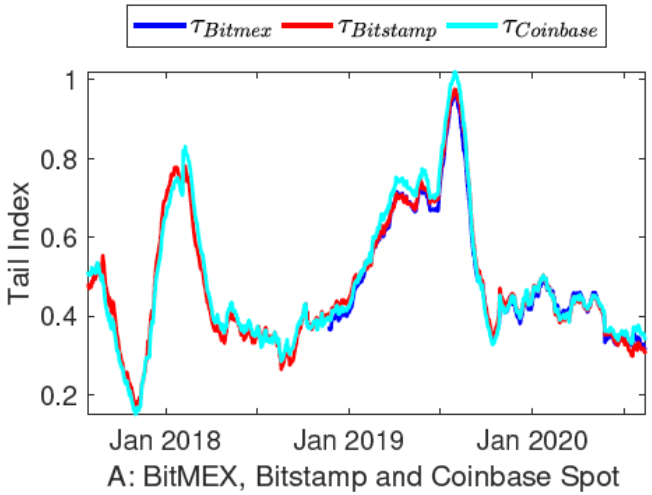
Consider results for 30-minute monitoring and hedge horizon of 5 days

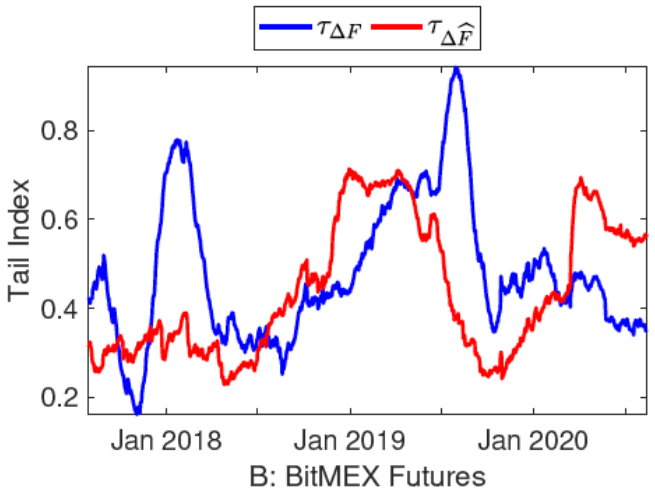
We estimate GED parameters based on 210 daily extreme values assumed to be maximum of $\left(\Delta\widehat{F}_{t+\Delta t}, \Delta\widehat{F}_{t+2\Delta t}, \cdots, \Delta\widehat{F}_{t+48\Delta t}\right)$

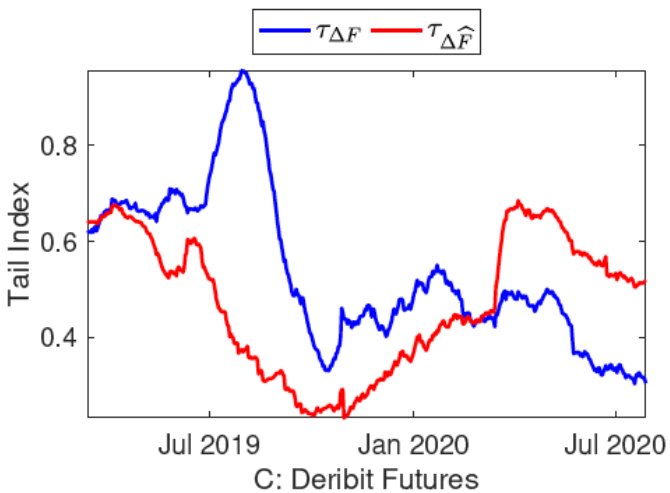


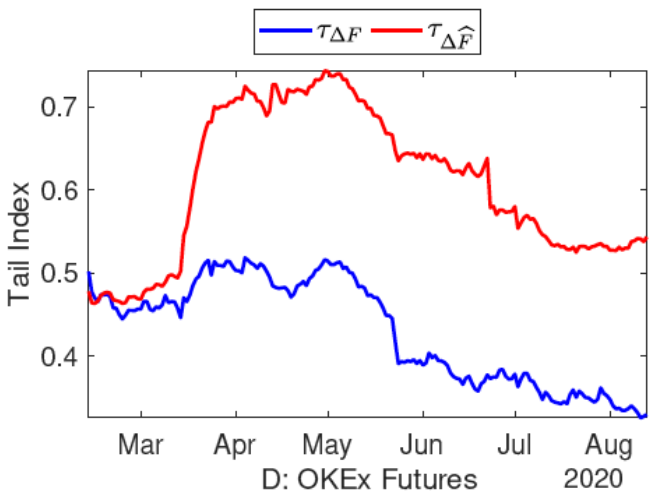
Most important parameter for optimal hedge ratio is right tail index

Rolling Estimation of Right Tail Index



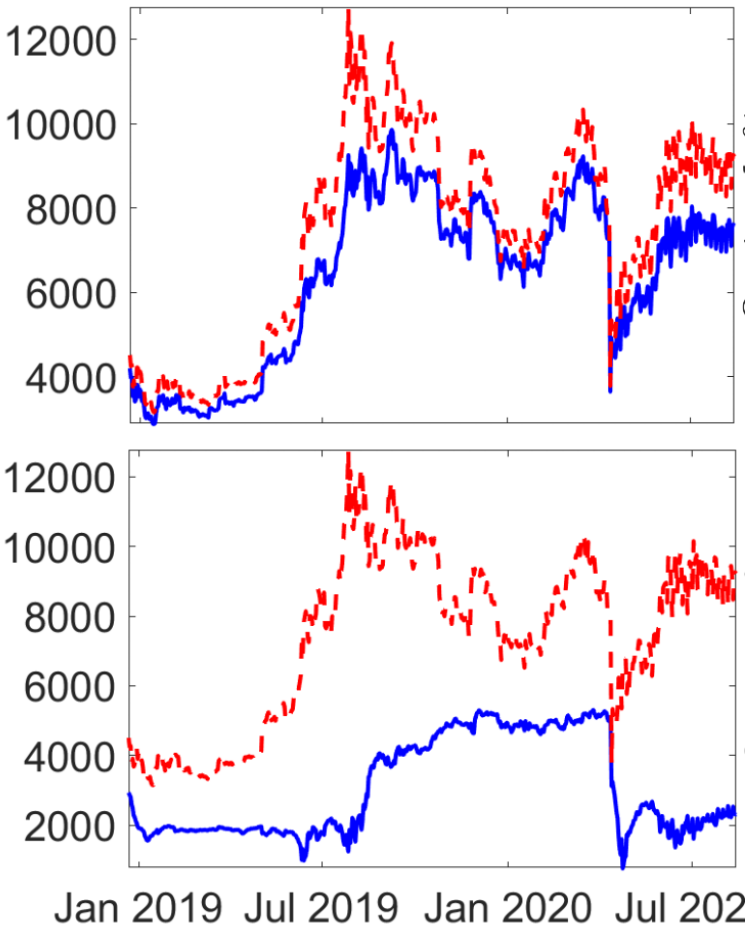


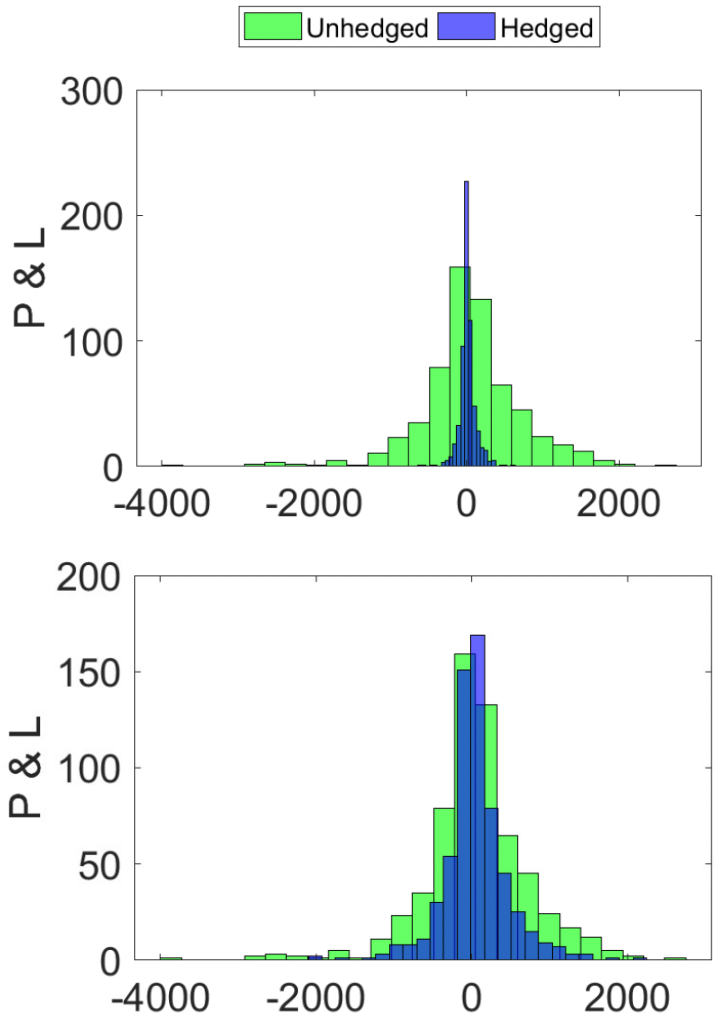




BitMEX Hedge Effectiveness

Rolling θ^* for 0, low and high γ





Summary So Far

- Right tail indices are similar for spot but very different for perpetuals
- Modelling inverse contract matters
- Margin mechanisms have a significant impact on optimal hedge ratios
- Different platforms provide varying levels of hedging effectiveness
- Default aversion reduces hedging and so decreases hedge efficiency
- Work in progress:
  - Theory – Model for m_t yielding analytic solution?
  - Empirical – Realistic values and n?
  - Need to include funding rate payments?