# Hedging with Bitcoin Futures: The Effect of Automatic Liquidation and Leverage Selection

Fields - CFI Workshop on Mathematical Finance and Cryptocurrencies

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# Overview

- We model the hedging of bitcoin spot with bitcoin perpetual or calendar futures on several centralised crypto derivatives exchanges
- Our hedging model incorporates leverage selection and auto-liquidations – novel microstructure, margining and clearing processes employed by 'self-regulated' CEXs
- The only regulated crypto derivatives exchange is the CME, which is not 24/7. Speculative activity on other CEXs is rife, so we introduce new measures of speculative activity which capture the novel microstructure of CEX trading
- We use the model and metrics to compare different CEX platforms and products according to (i) hedge efficiency and (ii) speculative activity

# Outline

1. Who Hedges Bitcoin – and How ?
2. Trading on Centralised Crypto Derivatives Exchanges
3. Speculation Metrics: Theory and Application
4. Hedging with Leverage Selection and Auto-Liquidations

# I. Who Hedges Bitcoin – and How?

- Institutional investors using bitcoin (BTC) as numeraire
- Companies whose profits are tied to BTC with expenditures in local fiat currency
    1. Metaverse developers
    2. Blockchain miners
    3. Crypto exchanges
    4. Market makers on centralised exchanges
    5. Crypto brokers and custodians

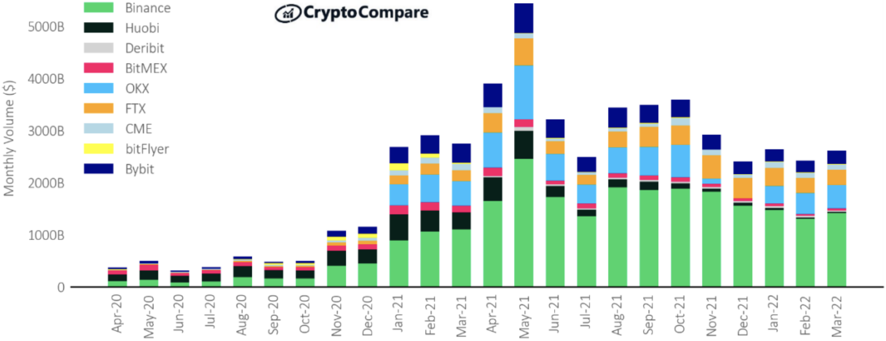
# Hedging Instruments: Calendar Futures

- Standard futures
  - Bitcoin is the underlying
  - Quoted, margined and settled in USD
  - E.G. CME contracts have 6 monthly and quarterly-cycle expires
  - Notional value is 5 or 0.1 bitcoins ;
- Direct futures
  - Bitcoin is the underlying
  - Quoted, margined and settled in stable coins (such as USDT)
  - E.G. Bybit direct USDT futures have notional value of 1 bitcoin
- Inverse futures
  - USD is the underlying
  - Quoted in USD per bitcoin, like standard futures
  - But margined and settled in BTC
  - E.G. Binance inverse futures have a notional value of 100 USD

# Perpetual Futures Contracts

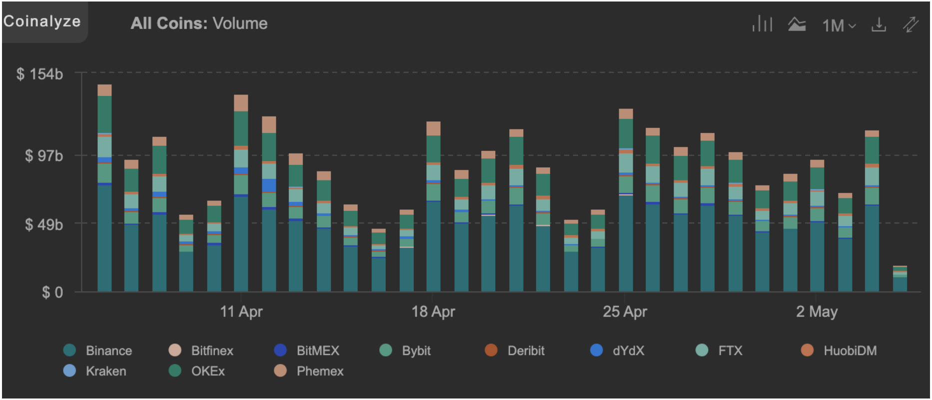
- Unique to crypto derivatives exchanges
- No maturity – perpetual price tied to spot price via funding payments
- Funding rate determines payment from long to short counterparty
- Funding rate is sum of small constant interest rate, plus a premium
- Premium is positive when perpetual price $>$ spot price
- Premium is negative when perpetual price $<$ spot price
- Much the most actively traded instrument on every crypto exchange

# II. Trading on Centralised Crypto Derivatives Exchanges




**Binance leads the derivative markets with 53.9% ($1.41tn) of total volumes in March.** This was followed by OKX (17.0% market share, $446bn) and Bybit (10.0% market share, $262bn).

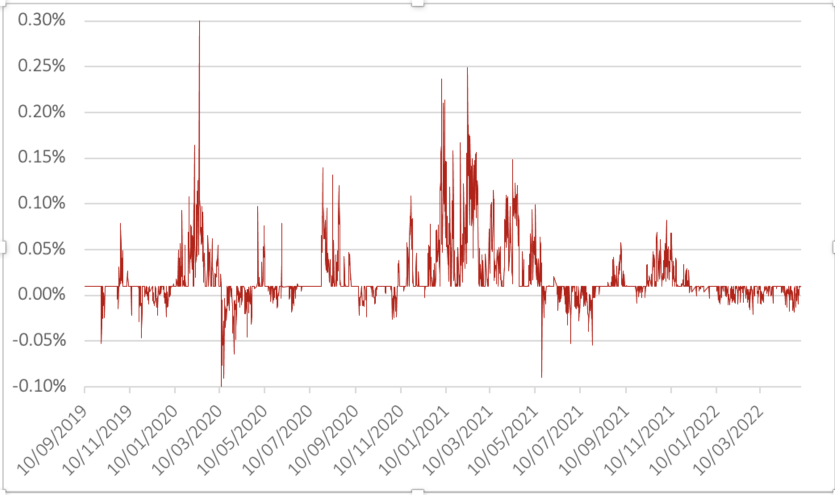
# Recent Daily Volumes



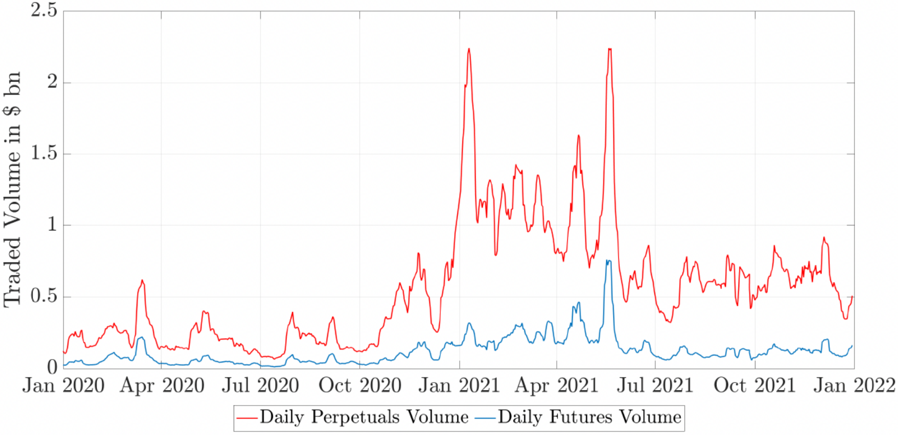

Data from coinanalyse.net

# Binance BTCUSDT Funding Rate

Data available here



# Comparison of Trading Volumes (Deribit)



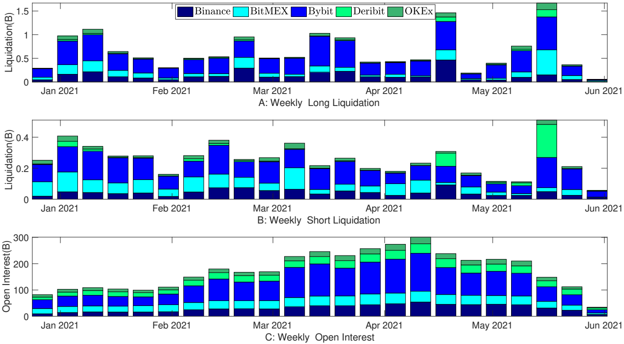

Taken from: Delta Hedging Bitcoin Options with a Smile

# Clearing Activities: Liquidation Protocol

- Users select leverage up to 100X (direct) or 125X (inverse)
- Initial margin is reciprocal of leverage, e.g. 20X leverage implies 5%
- Collateral account covers all positions of one user: balance is initial collateral used to open the account + realised PnL from exit of previous + unrealised PnL which is based on mark price
- For example, see Binance fair price marking mechanism
- Maintenance margin depends on position size and platform
- For example, on Binance for BTCUSDT the maintenance margin is between 0.4% and 2.5% of each position notional, this fraction increasing with the size of position
- Automatic liquidations are triggered when collateral falls below maintenance margin of any position

# Auto-Liquidations on USD Inverse Perpetuals ($Billion)

Almost $80 billion of positions on centralised exchanges were liquidated during 2021, an average of over $200 million per day




Auto-liquidations of long positions on BitMEX perpetual exceeded $800 million on 12 March 2020.

Data from coinanalyse.net

# Historical Frequency of Auto-Liquidation

## Bybit USD Inverse Perpetuals

- Suppose trader opens 1 long or short position, held over trading horizon (8h – 30d)
- Suppose no collateral added or subtracted from margin account
- Liquidation when marked losses exceed collateral
- Assume 100X leverage and maintenance margin rate 0.5% of nominal, so liquidation occurs when marked loss exceeds 0.5%

Results based on minute-level historical data from 1 July 2020 to 31 May 2021

| | Leverage | 8h | 1d | 5d | 15d | 30d |
|---|---|---|---|---|---|---|
| Short | 5X | 0.00% | 0.01% | 2.72% | 23.12% | 55.88% |
| | 20X | 4.71% | 16.54% | 53.18% | 77.11% | 91.35% |
| | 50X | 27.15% | 49.12% | 80.12% | 93.10% | 97.03% |
| | 100X | 50.35% | 70.86% | 89.52% | 96.44% | 98.22% |
| Long | 5X | 0.12% | 1.39% | 8.84% | 14.58% | 19.20% |
| | 20X | 7.04% | 18.45% | 38.78% | 51.48% | 54.33% |
| | 50X | 27.19% | 45.30% | 64.75% | 72.33% | 72.98% |
| | 100X | 47.41% | 63.00% | 78.34% | 82.97% | 83.21% |

# III. Speculation Metrics

- Turnover index is standard measure of speculation but this is does not capture the ability of users to choose their own leverage
- Level of leverage influences the probability of automatic counterparty liquidation, which may also result in automatic deleveraging of open positions
- We introduce a new speculation metric based on volume of liquidations
- We back-out another speculation metric called implied leverage using maximum drawdown and upbeat data
- We combine these two new metrics into a third measure, of trading aggressiveness which we can use to compare trading on different CEXs

# Turnover Index Garcia et al. (1986)

$$TI = \frac{\text{Trading Volume}}{\text{Open Interest}}$$

| | | USD Inverse Perpetuals | | | | | USDT Direct Perpetuals | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Binance | BitMEX | Bybit | Deribit | OKEx | Binance | Bybit | OKEx |
| Mean | Volume (B) | 1.03 | 0.49 | 1.20 | 0.20 | 0.25 | 2.75 | 0.22 | 0.29 |
| | OI (B) | 0.76 | 0.73 | 1.85 | 0.58 | 0.36 | 1.54 | 0.29 | 0.14 |
| | Short Liq. (M) | 1.05 | 1.48 | 2.87 | 0.48 | 0.39 | 7.90 | 1.50 | 1.54 |
| | Long Liq. (M) | 3.23 | 2.80 | 8.64 | 0.55 | 0.92 | 13.92 | 2.05 | 1.23 |
| | Liquidation (M) | 4.28 | 4.28 | 11.51 | 1.03 | 1.31 | 21.83 | 3.55 | 2.77 |
| | TI | 1.54 | 0.71 | 0.77 | 0.38 | 0.76 | 1.91 | 0.80 | 2.11 |
| Median | Volume (B) | 0.88 | 0.40 | 0.95 | 0.15 | 0.20 | 2.31 | 0.17 | 0.23 |
| | OI (B) | 0.70 | 0.75 | 1.86 | 0.58 | 0.34 | 1.61 | 0.30 | 0.14 |
| | Short Liq. (M) | 0.59 | 0.32 | 1.69 | 0.01 | 0.14 | 3.93 | 0.72 | 0.50 |
| | Long Liq. (M) | 1.23 | 0.37 | 4.08 | 0.02 | 0.16 | 5.08 | 1.05 | 0.49 |
| | Liquidation (M) | 2.07 | 1.33 | 6.72 | 0.05 | 0.52 | 11.67 | 2.34 | 1.45 |
| | TI | 1.24 | 0.57 | 0.58 | 0.28 | 0.60 | 1.54 | 0.62 | 1.66 |

The 4-hour mean and median of trading volumes and open interest (billion USD), and short, long and total liquidations (million USD). Bitcoin perpetuals are USD inverse and USDT direct across Binance, BitMEX, Bybit, Deribit and OKEx. The data period is from 1 Jan to 31 May 2021.

# Liquidation Index

New speculation index for crypto markets

$$\text{LI} = \text{Liquidation Index} = \frac{\text{Liquidation Volume}}{\text{Trading Volume}}$$

- Numeraire is total volume of auto-liquidations
- Exchanges (mis)report volumes of long and short liquidations, every 4 hrs

Average of 4-hourly Liquidation Indices between 1 Jan and 31 May 2021

| | USD Inverse Perpetuals | | | | | USDT Direct Perpetuals | | |
|---|---|---|---|---|---|---|---|---|
| | Binance | BitMEX | Bybit | Deribit | OKEx | Binance | Bybit | OKEx |
| LI (long) | 0.11% | 0.34% | 0.55% | 0.10% | 0.17% | 0.36% | 0.76% | 0.44% |
| LI (short) | 0.27% | 0.37% | 0.28% | 0.14% | 0.19% | 0.47% | 0.62% | 0.31% |
| LI (total) | 0.39% | 0.71% | 0.83% | 0.23% | 0.35% | 0.83% | 1.38% | 0.75% |

# Implied Leverage

## For a long position on a direct futures

- Leverage $\lambda_l^d$ used to open long position at time $0$ when perpetual price is $F_o$
- $\Rightarrow$ initial margin $\text{IM}_o = \frac{F_o}{\lambda_l^d}$
- Assume no additional collateral in/out of margin account
- Suppose maintenance margin rate is $\frac{1}{2}$ initial margin rate $\Rightarrow \text{MMR}_t = \frac{1}{2\lambda_l^d}$
- $\Rightarrow$ Collateral in margin account at time $t > 0$ is $\text{IM}_o - \text{MM}_t = \frac{F_o}{\lambda_l^d} - \frac{F_t}{2\lambda_l^d}$
- Max loss on long position in $[0, T]$ is $F_o - F_L$ where $F_L$ is low price
- Auto-liquidation occurs in $[0, T]$ if loss exceeds collateral, i.e. when:

$$F_o - F_L > \frac{1}{\lambda_l^d}\left(F_o - \frac{1}{2}F_L\right)$$

- Implied leverage is minimum value of $\lambda_l^d$ to prevent auto-liquidation
- Found by setting equality above $\Rightarrow \lambda_l^d = \frac{F_L}{2(F_o - F_L)} + 1$

# Implied Leverage

## Formulae for long and short positions on direct and inverse products

- Similar arguments for short positions on direct $d$ perpetuals, and long $l$ or short $s$ positions on inverse perpetuals $i$ yield the following:

$$\lambda_l^d = \frac{F_L}{2(F_o - F_L)} + 1,\ \lambda_s^d = \frac{F_H}{2(F_H - F_o)} - 1$$
$$\lambda_l^i = \frac{F_o}{2(F_o - F_L)} - 1,\ \lambda_s^i = \frac{F_o}{2(F_H - F_o)} + 1$$

Average implied leverages over all 4-hr internals from 1 Jan to 31 May 2021

| | USD Inverse Perpetuals | | | | | USDT Direct Perpetuals | | |
|---|---|---|---|---|---|---|---|---|
| | Binance | BitMEX | Bybit | Deribit | OKEx | Binance | Bybit | OKEx |
| $\lambda_l$ | 11.26 | 8.28 | 12.53 | 7.04 | 8.71 | 12.55 | 14.16 | 13.51 |
| $\lambda_s$ | 14.95 | 14.38 | 15.23 | 6.70 | 15.17 | 12.64 | 12.61 | 10.41 |

- Implied leverage like average leverage of all traders on exchange
- These are averages, but implied leverage time series values can reach 30% or so

# Aggressiveness Index

## Combines our leverage index and the implied leverage into a single new speculation index for crypto markets

$$\text{AI} = \text{Aggressive Index} = \frac{\sum_t [\lambda_{lt}\text{LLV}_t + \lambda_{st}\text{SLV}_t]}{\sum_t \text{TV}_t}$$

- $\text{TV}_t$ is total volume traded during time interval $t$
- $\text{LLV}_t$ and $\text{SLV}_t$ are total size of long and short auto-liquidations
- Aggregated over 4-hr intervals [frequency of data published by exchanges]
- Also decomposed into long and short indices....

Aggressive Indices for Period 1 Jan and 31 May 2021

| | USD Inverse Perpetuals | | | | | USDT Direct Perpetuals | | |
|---|---|---|---|---|---|---|---|---|
| | Binance | BitMEX | Bybit | Deribit | OKEx | Binance | Bybit | OKEx |
| AI (long) | 3.52% | 4.76% | 9.03% | 1.93% | 3.22% | 6.25% | 13.12% | 5.82% |
| AI (short) | 1.52% | 4.37% | 3.65% | 1.61% | 2.41% | 3.63% | 8.81% | 5.63% |
| AI | 5.04% | 9.13% | 12.69% | 3.54% | 5.63% | 9.89% | 21.92% | 11.45% |

# IV. Optimal Hedging in the Presence of Auto-Liquidations

- Here we only present theory for P&L formulation, and for direct perpetual, with USDT price $F_t$ but paper contains theory for returns formulation, and for both direct and inverse perpetuals
- Assume the hedger holds 1 bitcoin at time $t$ with USD spot price $S_t$ and shorts $\theta$ units of the perpetual to hedge spot price volatility over hedge horizon
- Suppose the platform monitors the margin at discrete intervals $\Delta t$ until the hedge horizon $t + N\Delta t$
- Standard hedging problem is to choose $\theta$ to minimize the variance of $\Delta h$, the P&L of the hedged portfolio P&L,

$$\sigma^2_{\Delta h}(\theta) = \mathbb{V}\mathrm{ar}\left[(S_{t+N\Delta t} - S_t) - \theta\left(F_{t+N\Delta t} - F_t\right)\right]$$

where we assume that USDT = USD

# Adding the Margin Constraint

- Margin constraint is represented by a positive number $\overline{m}$ – an upper constraint on the hedger's ability to avoid auto-liquidation
- In other words, $\overline{m}$ is the USDT collateral in the margin account, in excess of the minimum maintenance margin $m_0$
- Following Longin (1999), introduce the extreme price change operators:

$$\Delta_N^* S_t := \max_{1 \leq n \leq N} \Delta_n S_t,\ \Delta_N^* F_t := \max_{1 \leq n \leq N} \Delta_n F_t$$

  where $\Delta_n S_t := S_{t+n\Delta t} - S_t$ and $\Delta_n F_t := F_{t+n\Delta t} - F_t$

- A short position of $\theta$ on the perpetual makes a loss when $\Delta_N^* F_t > 0$ and the auto-liquidation occurs whenever a loss exceeds available collateral $\bar{m}$. Hence

$$\mathbb{P}\mathrm{rob}(\text{auto-liquidation}) = P(\overline{m}, \theta) := \mathbb{P}\mathrm{rob}\left(\theta \cdot \Delta_N^* F_t > \overline{m}\right)$$

- This probability decreases with $\overline{m}$ and increases with $\theta$ – i.e. greater financial reserves (larger $\overline{m}$) or less risk taking activities (smaller $\theta$) both reduce the chance of liquidation.

# Hedging Problem and Solution

Introduce a default aversion factor $\gamma$ to obtain final objective:

$$\min_{\theta>0} \quad \left\{\sigma^2_{\Delta h}(\theta) + \gamma\, \sigma^2_{\Delta S}\, P(\bar{m}, \theta)\right\}$$

Using the extreme value theorem we obtain an analytical expression for the optimal hedging strategy $\theta^*$ as a root of the equation:

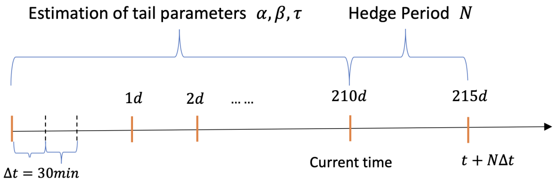
$$a(x)x^{-2} + x - b = 0,$$

where $b := \frac{\sigma^2_{SF}}{\sigma^2_F}$ and, setting $\nu := \frac{\sigma^2_S}{\sigma^2_F}$ and $\omega(x) := \frac{\tau}{\alpha}\left(\bar{m}x^{-1} - m_0 - \beta\right)$ we find

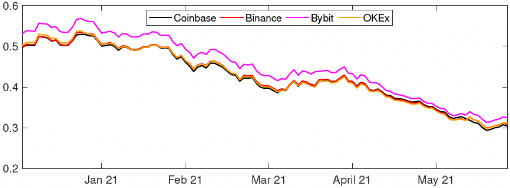
$$a(x) := \frac{\gamma\, \nu\, \bar{m}}{2\alpha}\, \exp\left[-\left(1+\omega(x)\right)^{-\frac{1}{\tau}}\right]\left(1+\omega(x)\right)^{-\frac{1}{\tau}-1}.$$

The parameters $\alpha$, $\beta$ and $\tau$ are respectively scale, location and tail index estimation parameters of the right tail of $\Delta^*_N F_t$. Most important parameter is right tail index $\tau$.

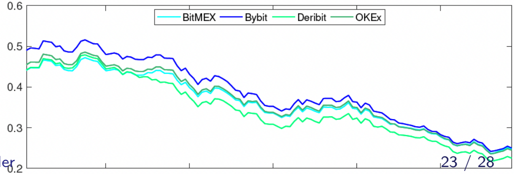
# Parameter Estimation



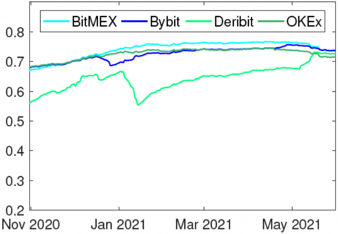

Estimation with 30-minute monitoring, 5-day horizon, rolling sample of 210 daily extreme values



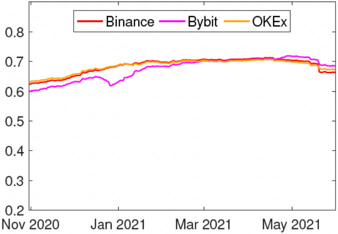

Panel A: Coinbase Spot and Direct Perpetuals Tail Index



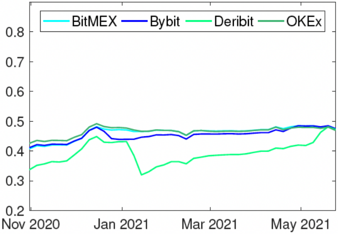
# Size of Optimal Hedge Position



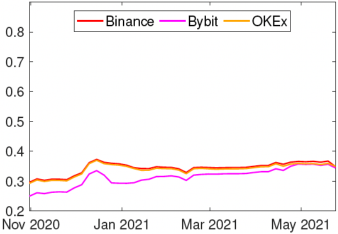

Panel A: (1d) Optimal $\theta_0^*$ for Inverse Perps




Panel B: (1d) Optimal $\theta_0^*$ for Direct Perps




Panel C: (5d) Optimal $\theta_0^*$ for Inverse Perps




Panel D: (5d) Optimal $\theta_0^*$ for Direct Perps

# Empirical Design

1. We measure the hedge effectiveness of the optimal hedge ratios using the standard Figlewski (1984) percentage reduction in portfolio variance

2. We also obtain the following approximation to the liquidation probability under the optimal strategy:

$$P(\overline{m}, \theta^*) \simeq 1 - \exp\left[-\left(1 + \omega(\theta^*)\right)^{-1/\tau}\right],$$

which follows from a standard extreme value theorem convergence result:

$$\mathbb{P}\left(\max_{1 \leq n \leq N} \Delta_n F \leq x\right) \quad \longrightarrow \quad \exp\left[-\left(1 + \tau\left(\frac{x - \beta}{\alpha}\right)\right)^{-1/\tau}\right],$$

3. Finally, the margin $m$ per contract is $\frac{m}{\theta^*}$ in USDT, so the margin rate per contract is $\frac{m}{\theta^* F}$ and the implied leverage under the optimal strategy is $\frac{\theta^* F}{m}$.

# Empirical Results

Table 3: Hedge Effectiveness, Liquidation Probability and Implied Leverage under the Optimal Hedging Strategy

| | | | Inverse Perpetuals | | | | | | | | | | | | Direct Perpetuals | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | BitMEX | | | Bybit | | | Deribit | | | OKEx | | | Binance | | | Bybit | | | OKEx | | |
| | $\overline{m}$ | $\gamma$ | 8h | 1d | 5d | 8h | 1d | 5d | 8h | 1d | 5d | 8h | 1d | 5d | 8h | 1d | 5d | 8h | 1d | 5d | 8h | 1d | 5d |
| HE(%) | 50% | 10 | 99.6 | 99.5 | 99.2 | 97.8 | 97.9 | 97.7 | 85.5 | 90.7 | 95.4 | 99.3 | 99.2 | 98.5 | 99.2 | 99.0 | 97.1 | 97.6 | 97.7 | 95.9 | 99.2 | 99.0 | 97.1 |
| | | 20 | 99.5 | 99.2 | 97.6 | 97.7 | 97.5 | 95.8 | 85.2 | 90.0 | 92.6 | 99.2 | 98.7 | 96.7 | 99.1 | 98.3 | 93.0 | 97.4 | 97.1 | 91.2 | 99.1 | 98.3 | 92.9 |
| | | 40 | 99.3 | 98.2 | 93.2 | 97.5 | 96.5 | 91.2 | 84.7 | 88.3 | 86.8 | 98.9 | 97.6 | 92.3 | 98.7 | 96.4 | 83.1 | 97.1 | 95.2 | 79.7 | 98.7 | 96.4 | 82.7 |
| | 20% | 10 | 99.2 | 97.5 | 86.9 | 97.3 | 95.7 | 84.7 | 84.4 | 87.2 | 79.3 | 98.7 | 96.8 | 86.3 | 98.5 | 95.9 | 79.7 | 96.9 | 94.7 | 75.7 | 98.5 | 95.9 | 79.2 |
| | | 20 | 98.3 | 93.3 | 72.1 | 96.2 | 91.4 | 70.0 | 82.8 | 81.8 | 64.2 | 97.6 | 92.5 | 72.3 | 97.2 | 90.1 | 58.2 | 95.6 | 88.3 | 52.9 | 97.1 | 89.9 | 57.5 |
| | | 40 | 95.4 | 82.8 | 53.3 | 93.1 | 81.0 | 51.6 | 78.9 | 70.4 | 46.6 | 94.5 | 82.5 | 54.0 | 93.7 | 76.7 | 34.1 | 91.7 | 73.4 | 29.6 | 93.6 | 76.2 | 33.4 |
| | 10% | 10 | 96.3 | 85.8 | 52.3 | 94.1 | 84.0 | 50.5 | 80.1 | 73.4 | 45.3 | 95.4 | 85.3 | 53.1 | 94.9 | 82.8 | 41.2 | 93.0 | 80.1 | 36.0 | 94.7 | 82.4 | 40.4 |
| | | 20 | 89.9 | 68.5 | 33.7 | 87.6 | 66.9 | 32.6 | 72.8 | 56.4 | 29.1 | 89.2 | 68.8 | 34.7 | 88.0 | 62.9 | 20.5 | 85.4 | 58.7 | 17.3 | 87.8 | 62.2 | 20.0 |
| | | 40 | 75.8 | 45.4 | 20.5 | 73.6 | 44.4 | 19.8 | 59.0 | 36.1 | 17.7 | 75.8 | 46.5 | 21.2 | 73.7 | 37.3 | 9.1 | 69.8 | 33.2 | 7.6 | 73.2 | 36.5 | 8.9 |
| P(%) | 50% | 10 | 0.1 | 0.3 | 0.7 | 0.1 | 0.3 | 0.6 | 0.1 | 0.2 | 0.5 | 0.1 | 0.3 | 0.6 | 0.1 | 0.4 | 1.2 | 0.1 | 0.4 | 1.2 | 0.1 | 0.4 | 1.2 |
| | | 20 | 0.1 | 0.3 | 0.6 | 0.1 | 0.3 | 0.5 | 0.1 | 0.2 | 0.4 | 0.1 | 0.3 | 0.5 | 0.1 | 0.4 | 0.9 | 0.1 | 0.4 | 0.9 | 0.1 | 0.4 | 0.9 |
| | | 40 | 0.1 | 0.3 | 0.4 | 0.1 | 0.2 | 0.4 | 0.1 | 0.2 | 0.3 | 0.1 | 0.2 | 0.4 | 0.1 | 0.3 | 0.6 | 0.1 | 0.3 | 0.6 | 0.1 | 0.3 | 0.6 |
| | 20% | 10 | 0.6 | 1.2 | 2.1 | 0.6 | 1.2 | 2.0 | 0.5 | 1.0 | 1.6 | 0.5 | 1.2 | 2.1 | 0.6 | 1.4 | 2.7 | 0.6 | 1.4 | 2.5 | 0.6 | 1.4 | 2.7 |
| | | 20 | 0.5 | 1.0 | 1.1 | 0.5 | 0.9 | 1.1 | 0.4 | 0.8 | 0.8 | 0.5 | 0.9 | 1.1 | 0.5 | 1.1 | 1.3 | 0.5 | 1.1 | 1.1 | 0.5 | 1.1 | 1.2 |
| | | 40 | 0.4 | 0.6 | 0.5 | 0.4 | 0.6 | 0.5 | 0.3 | 0.5 | 0.3 | 0.4 | 0.6 | 0.5 | 0.4 | 0.7 | 0.4 | 0.4 | 0.6 | 0.3 | 0.4 | 0.7 | 0.4 |
| | 10% | 10 | 1.5 | 2.4 | 2.0 | 1.5 | 2.3 | 1.9 | 1.2 | 1.8 | 1.4 | 1.5 | 2.4 | 2.1 | 1.5 | 2.6 | 2.0 | 1.5 | 2.5 | 1.6 | 1.5 | 2.6 | 1.9 |
| | | 20 | 1.1 | 1.3 | 0.7 | 1.1 | 1.2 | 0.6 | 0.9 | 0.9 | 0.5 | 1.1 | 1.3 | 0.7 | 1.1 | 1.4 | 0.5 | 1.1 | 1.2 | 0.4 | 1.1 | 1.3 | 0.5 |
| | | 40 | 0.7 | 0.5 | 0.2 | 0.6 | 0.5 | 0.2 | 0.5 | 0.3 | 0.1 | 0.7 | 0.5 | 0.2 | 0.7 | 0.5 | 0.1 | 0.6 | 0.4 | 0.1 | 0.7 | 0.5 | 0.1 |
| L. | 50% | 10 | 1.9 | 1.9 | 1.8 | 1.9 | 1.9 | 1.8 | 1.7 | 1.7 | 1.6 | 1.9 | 1.8 | 1.7 | 1.9 | 1.8 | 1.6 | 1.9 | 1.9 | 1.7 | 1.9 | 1.8 | 1.7 |
| | | 20 | 1.9 | 1.9 | 1.7 | 1.9 | 1.8 | 1.6 | 1.7 | 1.7 | 1.5 | 1.8 | 1.8 | 1.6 | 1.8 | 1.7 | 1.5 | 1.9 | 1.8 | 1.4 | 1.8 | 1.8 | 1.5 |
| | | 40 | 1.9 | 1.8 | 1.5 | 1.8 | 1.7 | 1.4 | 1.7 | 1.6 | 1.3 | 1.8 | 1.7 | 1.4 | 1.8 | 1.6 | 1.2 | 1.8 | 1.6 | 1.1 | 1.8 | 1.6 | 1.1 |
| | 20% | 10 | 4.6 | 4.2 | 3.1 | 4.5 | 4.1 | 3.1 | 4.1 | 3.7 | 2.7 | 4.5 | 4.1 | 3.1 | 4.4 | 4.0 | 2.7 | 4.5 | 4.0 | 2.6 | 4.5 | 4.0 | 2.7 |
| | | 20 | 4.4 | 3.7 | 2.3 | 4.3 | 3.6 | 2.3 | 3.9 | 3.2 | 2.0 | 4.2 | 3.6 | 2.3 | 4.2 | 3.4 | 1.7 | 4.2 | 3.4 | 1.6 | 4.2 | 3.4 | 1.7 |
| | | 40 | 3.9 | 2.9 | 1.5 | 3.8 | 2.9 | 1.5 | 3.4 | 2.5 | 1.3 | 3.8 | 2.9 | 1.6 | 3.8 | 2.6 | 0.9 | 3.8 | 2.4 | 0.8 | 3.8 | 2.6 | 0.9 |
| | 10% | 10 | 8.1 | 6.2 | 3.0 | 7.9 | 6.1 | 3.0 | 7.1 | 5.3 | 2.5 | 7.9 | 6.2 | 3.1 | 7.8 | 5.9 | 2.3 | 7.8 | 5.6 | 2.0 | 7.8 | 5.8 | 2.2 |
| | | 20 | 6.8 | 4.4 | 1.8 | 6.7 | 4.3 | 1.8 | 5.9 | 3.6 | 1.5 | 6.7 | 4.4 | 1.9 | 6.6 | 3.9 | 1.0 | 6.4 | 3.6 | 0.9 | 6.6 | 3.8 | 1.0 |
| | | 40 | 5.0 | 2.6 | 1.0 | 4.9 | 2.5 | 1.0 | 4.2 | 2.1 | 0.9 | 5.1 | 2.7 | 1.1 | 4.9 | 2.0 | 0.4 | 4.6 | 1.8 | 0.4 | 4.9 | 2.0 | 0.4 |

This table reports the hedge effectiveness (17) (HE(%) in the top panel), the liquidation probability (12) (P(%) in the middle panel) and the implied leverage (19) (L. in the bottom panel), all under the optimal hedging strategy (14). We set $\Delta t = 1$ minute and consider three margin constraint levels ($m = 10\%, 20\%, 50\%$), three loss aversion levels ($\gamma = 10, 20, 40$) and three hedge horizon levels ($N\Delta t =$ 8h, 1d, 2d).

# Conclusions

1. The size of the optimal hedge position decreases with the hedge horizon and also with both tighter margin constraint and a greater liquidation loss aversion, justifying the incorporation of these two features in the analysis;
2. Both inverse and direct products are highly effective hedging instruments, with effectiveness exceeding 90% provided the margin level is not too low, and inverse perpetuals have a slight advantage over their direct counterparts;
3. The optimal strategy yields a highly efficient and robust hedge with a hedge ratio of around 0.7 for 1-day horizons but only 0.4 for 5-day horizons because the liquidation probability increases with horizon;
4. The optimal strategy is very conservative: the hedger uses a low implied leverage $< 5$X with a very low liquidation probability $< 1\%$;
5. No significant differences in hedge effectiveness of different CEXs, but Deribit has least speculative activity and Bybit has the most.

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