

Bitcoin Derivatives

Financial Economics Meeting on Crisis Challenges

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Outline

1. Introduction to Bitcoin Derivatives
2. Margin Mechanisms on Unregulated Exchanges
3. Bitcoin Price and Volatility Transmissions from Unregulated Perpetuals
4. Bitcoin Options and Variance Swaps

1. Introduction to Bitcoin Derivatives

Bitcoin Spot Price: 1 Jan 2018 to 10 June 2021



Falls correlate with CME futures expires \Rightarrow Painting the tape?

Speculation on Bitcoin

- Who speculates?
 - Traditional retail investor base
 - Reddit
 - Robinhood
 - More recently, institutional investors
 - US banks allowed custody of bitcoin since June 2020
 - Specialised traders and brokers – GSG, [XBTO](#), BitWise etc.
- Which derivatives?
 - Perpetuals on unregulated exchanges attract different types of traders
 - Binance: coin rebates attracts large positions
 - Bybit: fee rebates encourage small traders
 - CME futures are not traded 24/7, speculation mostly painting the tape
 - *Follow* prices on unregulated exchanges (Alexander and Heck, 2020)
 - Most speculative produce are **leveraged tokens** and **variance swaps**
 - UP/DOWN tokens traded on centralised exchanges (esp. Binance)
 - Variance swaps are traded on-chain (e.g. by GSR)

Who Hedges Bitcoin?

- **Bitcoin miners:**
 - About 18.5 million coins have been mined
 - BTC supply stops in 2140, capped at 21 million
 - Currently, 6.25 coins mined every 10 mins
 - About 150,000 BTC mined since 1 Jan 2021
 - 150,000 BTC worth almost \$10 trillion at height of market
- **Option market makers:**
 - Open interest on Deribit bitcoin options currently \sim \$6 billion
 - Relative volumes \Rightarrow market makers delta hedge with perpetuals
- **Other players:**
 - Leveraged token market makers hedge with perpetuals
 - Variance swap issuers hedge with options

Standard, Direct and Inverse Bitcoin Futures

- **Standard bitcoin futures**

- Bitcoin is the underlying asset
- E.G. CME contracts have notional value 5 or 0.1 BTC
- Quoted, margined and settled in USD
- Standard expires, but do not trade 24/7

- **USDT *direct* futures**

- Bitcoin is the underlying asset
- E.G. Bybit direct USDT futures have notional value of 1 BTC
- Quoted, margined and settled in a stablecoin like USDT
- Non-standard expires, traded 24/7

- **USD *inverse* futures**

- Quoted in USD per bitcoin, like direct futures
- But with USD notional and margined and settled in BTC
- E.G. Binance inverse futures have a notional value of \$100
- Non-standard expires, traded 24/7

Leverage and Minimum Margin Rates

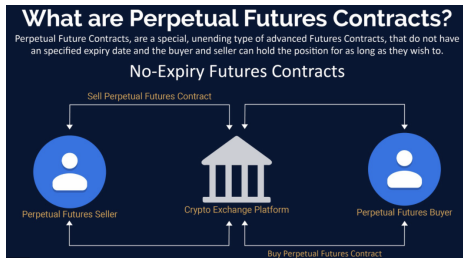
| 125x USDT-Margined Perpetual Contract (BTCUSDT) | | |
|---|----------|---------------------|
| Position (Notional Value in USDT) | Leverage | Initial Margin Rate |
| 0 < Position ≤ 50,000 | 125x | 0.80% |
| 50,000 < Position ≤ 250,000 | 100x | 1.00% |
| 250,000 < Position ≤ 1,000,000 | 50x | 2.00% |
| 1,000,000 < Position ≤ 10,000,000 | 20x | 5.00% |
| 10,000,000 < Position ≤ 20,000,000 | 10x | 10.00% |
| 20,000,000 < Position ≤ 50,000,000 | 5x | 20.00% |
| 50,000,000 < Position ≤ 100,000,000 | 4x | 25.00% |
| 100,000,000 < Position ≤ 200,000,000 | 3x | 33.30% |
| 200,000,000 < Position ≤ 300,000,000 | 2x | 50.00% |
| 300,000,000 < Position ≤ 500,000,000 | 1x | 100.00% |

| BTCUSD Perpetual Contracts | | |
|----------------------------|---------------------|--|
| Leverage | Initial Margin Rate | BTCUSD Position (Notional Value in BTC) |
| 125x | 0.8% | Max Position 10 |
| 100x | 1.0% | Max Position 20 |
| 50x | 2.0% | Max Position 30 |
| 20x | 5.0% | Max Position 50 |
| 10x | 10.0% | Max Position 100 |
| 5x | 20.0% | Max Position 200 |
| 4x | 25.0% | Max Position 400 |
| 3x | 33.3% | Max Position 1,000 |
| 2x | 50.0% | Above 1,000 |
| 1x | 100.0% | Above 1,000 |

- Maximum leverage (minimum initial margin rate) decreases with position size
- Maintenance margin rate 50% minimum initial margin rate per notional value

Types of Margin Mode

Perpetual Futures Contracts



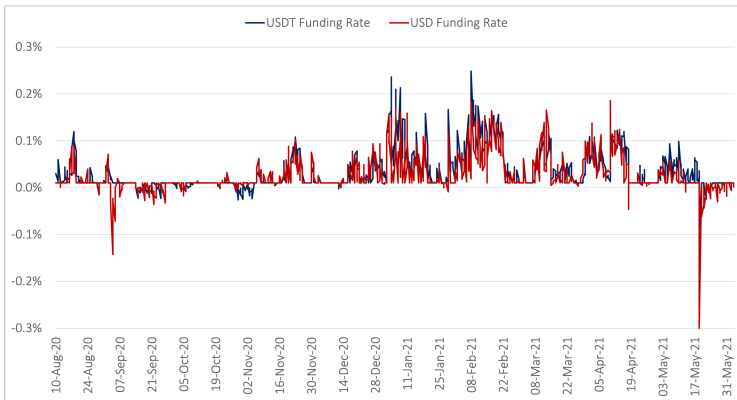
Sales pitch for Binance perptuals

Binance BTCUSDT Direct Perpetual

Binance BTCUSD Inverse Perpetual

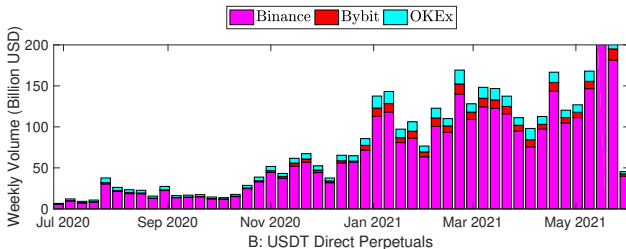
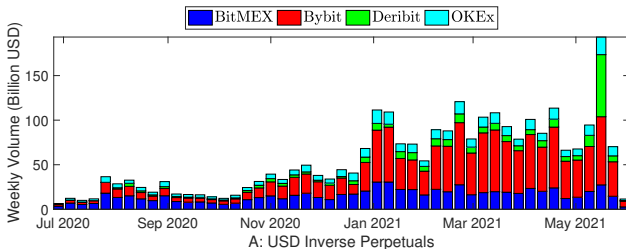
What Links Perpetual Prices to Spot Prices?

Funding Rates for Binance Perpetuals

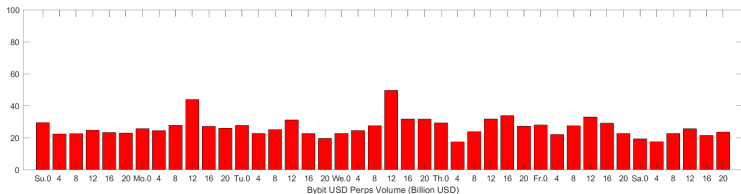
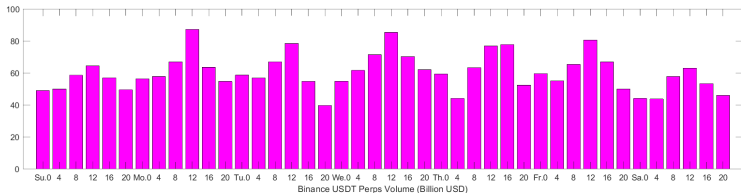


Funding Rate Calculation

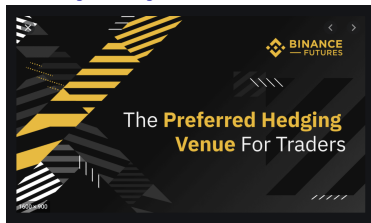
Evolution of Trading Volumes on Perpetuals



Time-of-Day Volume Patterns \Rightarrow Different Traders



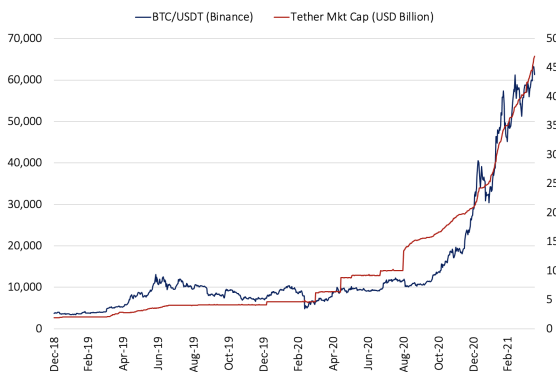
Bye Bye Binance?



“Three Reasons Why” and a **multitude** of reasons why not

- Unscrupulous business model
- Data fraud
- Total disregard for any regulation
- Sinister outages
- Now completely dominating the crypt market
- Tether!

Binance and Tether (and Bitfinex)



- Binance top of [tether rich list](#) – nearly \$17 billion in hot wallet
- Other identified exchange wallets together hold just over \$23 billion
- Tether market cap now over \$62 billion – other \$22 billion??

2. Margin Mechanisms on Unregulated Exchanges

- Unregulated exchanges issue no margin calls
 - Liquidations are **automatically triggered** when marked loss exceeds the collateral in the margin account
- Settlements are based on an **index price** – usually a simple average of BTC rates across the main spot exchanges
 - Many exchanges employ a **fair price marking** mechanism to mitigate auto-liquidations triggered by sudden spikes in index prices:

$$\text{Fair Mark Price} = \text{Index Price} \times (1 + \text{Funding Basis})$$

$$\text{Funding Basis} = \text{Funding Rate} \times (\text{Time to Funding} / \text{Funding Interval})$$

- The mark price is only used for unrealised P&L
- Funding rates are too small to impact the frequency of auto-liquidations
 - Except in crashes like 12-13 March 2020 or 11-19 May 2021

Insurance Fund and Auto Deleveraging (ADL)

BitMEX Trade • Contracts • References • API • Register • Log In

Auto Deleveraging

- Overview of Auto-Deleveraging (ADL)
- ADL Priority Deleveraging Ranking
 - Priority Ranking Calculation
 - More Information

Overview of Auto-Deleveraging (ADL)

When a trader's position is liquidated, the position is taken over by the BitMEX liquidation engine. If the liquidation cannot be filled by the time the mark price reaches the bankruptcy price, the ADL system automatically deleverages opposing traders' positions by profit and leverage priority.

The price at which a traders' positions are closed out is the bankruptcy price of the initial liquidated order.

ADL Priority Deleveraging Ranking

At all times, your position in the queue is shown by an indicator. This indicator represents your priority in the queue in 20% increments:

In the above example, all 'lights' are lit, which would mean your position is in the top percentile. In the case of a liquidation that is not able to be caught in the market, you may be deleveraged.

The Insurance Fund is used to prevent ADL. If it is depleted for a given contract, ADL will occur.

If you are deleveraged, you will be sent a notification. Open orders will be cancelled and you are free to re-enter.

Priority Ranking Calculation

Deleveraging priority is calculated by profit and leverage. More profitable and higher leveraged traders are deleveraged first.

The ranking calculation is as follows:

$$\text{Ranking} = \text{PNL Percentage} * \text{Effective Leverage} \quad (\text{if PNL percentage} > 0) \\ = \text{PNL Percentage} / \text{Effective Leverage} \quad (\text{if PNL percentage} < 0)$$

where

$$\text{Effective Leverage} = \text{abs}(\text{Mark Value}) / (\text{Mark Value} - \text{Bankrupt Value})$$
$$\text{PNL percentage} = (\text{Mark Value} - \text{Avg Entry Value}) / \text{abs}(\text{Avg Entry Value})$$
$$\text{Mark Value} = \text{Position Value at Mark Price}$$
$$\text{Bankrupt Value} = \text{Position Value at Bankruptcy Price}$$
$$\text{Avg Entry Value} = \text{Position Value at Average Entry Price}$$

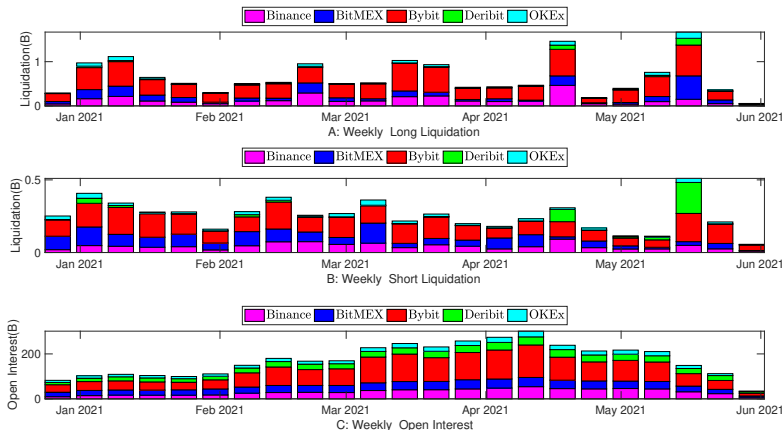
Historical Frequency of Auto-Liquidation on Bybit

- Suppose trader opens a long or short position
- Suppose leverage is 5X, 20X, 50X or 100X
- So initial margin rate is 20%, 5%, 2% or 1%
- Assume maintenance margin rate is half the initial margin
- What is probability of auto-liquidation if position is held for 8h – 30d?

Bybit USD Inverse Perpetuals 1-min data, 1 July 2020 to 31 May 2021

| | Leverage | 8h | 1d | 5d | 15d | 30d |
|-------|----------|--------|--------|--------|--------|--------|
| Long | 5X | 0.25% | 1.34% | 7.49% | 15.38% | 22.29% |
| | 20X | 18.84% | 35.56% | 56.48% | 65.12% | 66.17% |
| | 50X | 45.05% | 61.04% | 76.98% | 82.16% | 82.43% |
| | 100X | 64.19% | 75.57% | 85.12% | 87.89% | 87.94% |
| Short | 5X | 0.65% | 4.95% | 31.34% | 60.44% | 80.98% |
| | 20X | 20.62% | 41.73% | 75.14% | 91.00% | 96.51% |
| | 50X | 49.71% | 70.35% | 89.19% | 96.35% | 98.17% |
| | 100X | 69.41% | 83.39% | 93.48% | 97.71% | 98.85% |

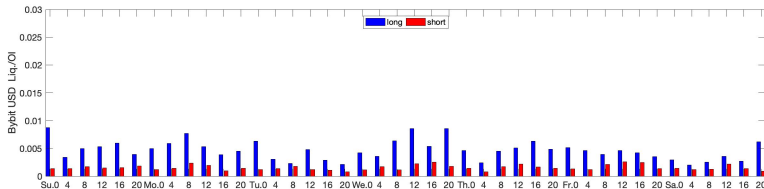
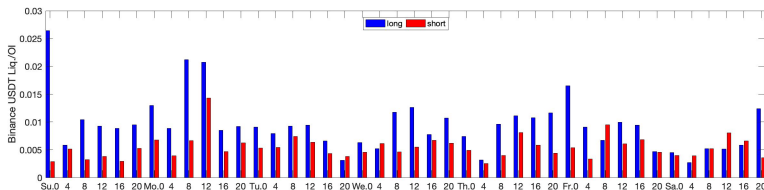
Auto-Liquidations on USD Inverse Perpetuals (\$billion)



4-hourly data from coinanalyse.net

Time Pattern of Auto-Liquidations?

Auto-liquidations as % OI on Binance and Bybit in 4hr time buckets



Comparison of ADL and Turnover

Turnover Index Garcia et al. (1986) \Rightarrow Speculation Metric

$$TI = \frac{\text{Trading Volume}}{\text{Open Interest}}$$

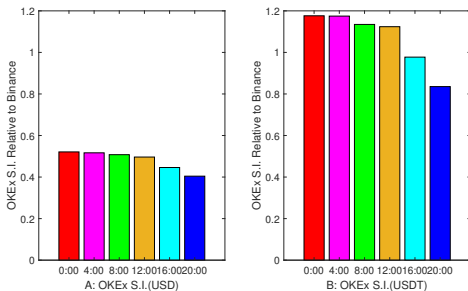
| | | USD Inverse Perpetuals | | | | | USDT Direct Perpetuals | | |
|------|-----------------|------------------------|--------|-------|---------|------|------------------------|-------|------|
| | | Binance | BitMEX | Bybit | Deribit | OKEx | Binance | Bybit | OKEx |
| Mean | Volume (B) | 1.03 | 0.49 | 1.20 | 0.20 | 0.25 | 2.75 | 0.22 | 0.29 |
| | OI (B) | 0.76 | 0.73 | 1.85 | 0.58 | 0.36 | 1.54 | 0.29 | 0.14 |
| | Short Liq. (M) | 1.05 | 1.48 | 2.87 | 0.48 | 0.39 | 7.90 | 1.50 | 1.54 |
| | Long Liq. (M) | 3.23 | 2.80 | 8.64 | 0.55 | 0.92 | 13.92 | 2.05 | 1.23 |
| | Liquidation (M) | 4.28 | 4.28 | 11.51 | 1.03 | 1.31 | 21.83 | 3.55 | 2.77 |
| | TI | 1.54 | 0.71 | 0.77 | 0.38 | 0.76 | 1.91 | 0.80 | 2.11 |

The 4-hour trading volumes and open interest (billion USD), and short, long and total liquidations (million USD). Bitcoin perpetuals are USD inverse and USDT direct across Binance, BitMEX, Bybit, Deribit and OKEx. The data period is from 1 Jan to 31 May 2021.

Data acquired from www.coinalyze.net.

OKEx Turnover Index \Rightarrow Chinese Traders

- Turnover on OKEx, relative to Binance, for six 4-hour intervals
- Data averaged over period 1 Jan to 31 May 2021



- Highest turnover 00:00 – 08:00 UTC [08:00 – 16:00 in China]
- Lowest turnover volumes 16:00 – 24:00 UTC [00:00 – 08:00 in China]

3. Price Discovery and Volatility Transmissions

- Numerous recent papers on bitcoin price discovery
 - An exception is Alexander et al. (2020a) examining ether
 - Most focus on the role of CME bitcoin futures
 - Alexander and Heck (2020) and Alexander et al. (2020b) highlight the role of **unregulated** exchanges such as BitMEX
- But crypto markets change extremely rapidly
 - Not only through innovations such as DeFi and NFTs
 - And the Chinese government ... and Elon Musk
 - But also due to actions (or inactions) of market regulators
 - Binance behaves as if immune to any regulatory or legal constraints
- Volatility transmissions almost all from Binance perptuals
 - And the most influential Binance product is its **USDT perpetual**
 - Is trading on this product driving **tether grants**?

Latest Results on Price Discovery

Let \mathbf{p}_t be the $n \times 1$ vector of cointegrated log prices at time t and let $z_t = \beta^T \mathbf{p}_t$ denote their deviations from long-run equilibrium. Then the VECM is:

$$\Delta \mathbf{p}_t = \alpha + \sum_{i=1}^{q-1} \Gamma_i \Delta \mathbf{p}_{t-i} + \delta z_{t-1} + \mathbf{e}_t,$$

where \mathbf{e}_t are serially uncorrelated innovations with zero mean and covariance matrix Ω and δ captures reactions to transitory equilibrium deviations. Inverting and integrating gives:

$$\mathbf{p}_t = \mathbf{p}_0 + \Psi(1) \sum_{j=1}^t \mathbf{e}_j + \Psi^*(L) \mathbf{e}_t$$

where $\Psi(1)$ i.e. the sum of the MA coefficients in the inversion of the AR, has identical rows which we denote ψ . Then the scalar $\psi \mathbf{e}_t$ is the long-term **common efficient price** which has variance $\psi \Omega \psi^T$ and $\Psi^*(L) \mathbf{e}_t$ captures the transitory components

Price Discovery Metrics

When new information arrives to the network the Hasbrouck (1995) information share asks *When new information enters the network, what proportion of the total price innovation originates on each product?* It is measured by its relative contribution to the variance of the common efficient price, i.e.:

$$IS_i = \frac{([\psi\mathbf{M}]_i)^2}{\psi\mathbf{\Omega}\psi^T} \quad \text{for } i = 1, \dots, N,$$

where \mathbf{M} is the lower triangular matrix of the Cholesky decomposition of $\mathbf{\Omega}$ and $[\psi\mathbf{M}]_i$ is the i -th entry of $\psi\mathbf{M}$. Various improvements of the IS have been proposed. We use the **generalised information share** of Lien and Shrestha (2009)

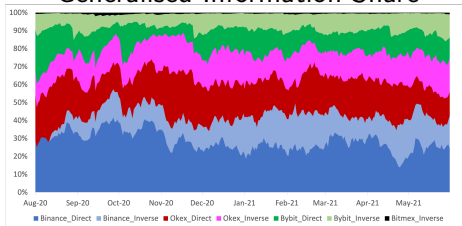
We also use the same VECM to compute the **component share** of Gonzalo and Granger (1995) which *assigns shares of the permanent, long-memory components of the common efficient price*. This measures the impact of each product on long-term price formation.

Procedure

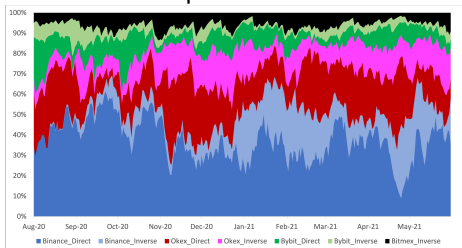
- Day-by-day estimation of various n -dimensional vector error correction models
 - Generalised Information Share (GIS)
 - Component Share (CS)
- 1-min OHLC data \Rightarrow traded BTC prices on different products:
 - Spot: Coinbase, Kraken, Bitstamp
 - Direct (tether) perpetuals: Binance, OKEEx, Bybit
 - Inverse Perpetuals: Binance, OKEEx, Bybit, BitMex
- Data from 1 August 2020 to 31 May 2020
- Exponential smoothing aids visual displays
- Daily time series of GIS and CS for each product

Results: Information Flows between Perpetuals

Generalised Information Share



Component Share



High-Frequency Volatility Spillover

Alexander, Heck and Kaeck (2021) The Role of Binance in Bitcoin Volatility Transmission

| Exchanges | |
|------------------------|-----------------------|
| Spot | Perpetuals |
| Bitstamp ^{\$} | Binance ^{\$} |
| Coinbase ^{\$} | Bybit ^{\$} |
| Kraken ^{\$} | Binance ^T |
| Binance ^T | |
| Huobi ^T | |

Research Questions

- Where does bitcoin volatility emerge?
 - Which exchange is the main emitter?
- To which exchanges does volatility transmit?
 - Which exchanges receive the most?
- Do volatility flows change over the course of the day?

Perpetual Specifications and Data

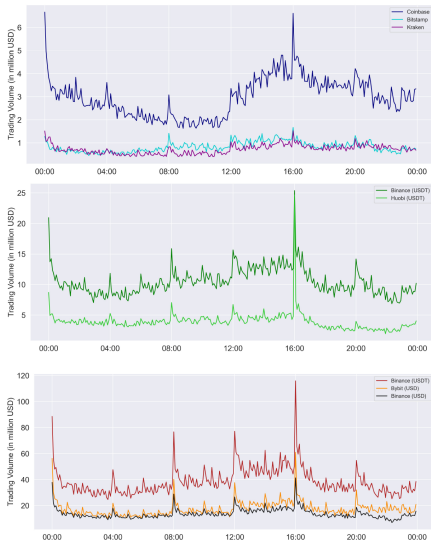
| | USD Contracts | | USDT Contracts |
|---------------------------|---------------|----------|----------------|
| | Binance | Bybit | Binance |
| Type | Inverse | Inverse | Direct |
| Contract Size | 100 USD | 1 USD | 0.001 BTC |
| Initial Margin Rate | > 0.8%* | 1% | > 0.8%* |
| Settlement Currency | BTC | BTC | USDT |
| Trading Days | 24/7 | 24/7 | 24/7 |
| Funding Frequency | 8 hrs | 8 hrs | 8 hrs |
| Fees (maker/taker) in bps | 1/5 | -2.5/7.5 | 2/4 |
| Tick Size | 0.1 USD | 0.5 USD | 0.01 USDT |

Note: Only showing the main perpetual contracts analysed. *Margin rates on Binance increase with notional value of position.

Data filtering in Alexander et al. (2021b):

- Second-by-second data from 1 January to 31 March 2021
- Calculate realised volatility at 5-minute frequency
- Winsorizing top 0.05% to reduce influence of extreme outliers
- Pre-averaging to reduce microstructure noise

Intra-day Volume Patterns (UTC)



Note: The figure shows the intraday pattern of five-minute trading volume (in million USD) for USD spot pairs (upper graph), USD spot pairs (middle graph) and perps (lower graph), measured as the median five-minute volume over the period from 1 January to 31 March 2021. All times are in UTC.

- Volume per 5-minute time bucket, averaged between 1 Jan and 31 Mar 2021. Spot USD (top), Spot USDT (middle) and Perpetuals (bottom)
- Coinbase trading similar to FX markets
→ increased volume between 12:00 UTC (7:00 EST) and 21:00 UTC (16:00 EST)
- Spikes at funding times of perpetuals, especially midnight and 16:00 UTC

Intra-day Realised Volatility Patterns (UTC)



Note: The figure shows the intraday pattern of 5-minute realised volatility (in million USD) for USD spot pairs (upper graph) as well as USDT spot pairs and perpetuals (lower graph), measured as the average five-minute realised volatility over the period from 1 January to 31 March 2021. All times are in UTC.

- Realised volatility per 5-minute time bucket, averaged between 1 Jan and 31 Mar 2021. Spot USD (top), and perpetuals (bottom)
- Spot results (not shown below):
Coinbase → Bitstamp and Kraken
(weaker) Bitstamp → Kraken
- Spikes on perpetuals, especially midnight and 16:00 UTC

Vector Logarithmic Multiplicative Error Model

Basic specification for 5-min realised volatilities of 6 exchanges, x_t :

$$x_t = \mu_t \odot \varepsilon_t$$
$$\log \mu_t = \mathbf{w} + \mathbf{A} \log x_{t-1} + \mathbf{B} \log \mu_{t-1}$$

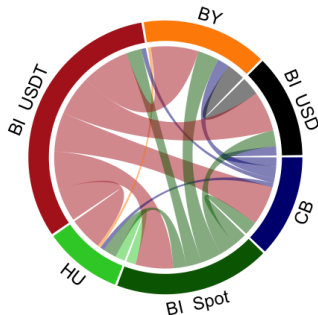
- Implicitly guarantees non-negativity of realised volatilities
- Decomposes realised volatility into Hadamard product of conditional mean and error term with unit mean and a distribution with non-negative support
- Log conditional mean is autoregressive \Rightarrow long-term effects \mathbf{B}
- Dependence on lagged observations $\log x_{t-1} \rightarrow$ short-term spillovers, \mathbf{A}
- Add asymmetric response component to capture leverage effect
- Also use an extension to capture zeros in high-frequency time series
- Also use dummies to investigate time-zone effects

Results – Main Instruments

| | Coinbase | Binance ^S | Huobi | Binance ^T | Bybit | Binance ^{\$} |
|------------------|-----------------------|----------------------|----------------------|----------------------|----------------------|-----------------------|
| CB | 0.2108 | −0.0895 | 0.0352 ^{ns} | 0.2548 | 0.0088 ^{ns} | −0.0173 ^{ns} |
| BI ^S | −0.0164 ^{ns} | 0.0742 | 0.0626 | 0.2439 | 0.0105 ^{ns} | 0.0223 ^{ns} |
| HU | −0.0282 | −0.0851 | 0.2720 | 0.2177 | 0.0198 | −0.0108 ^{ns} |
| BI ^T | −0.0307 | −0.0995 | 0.0247 ^{ns} | 0.4702 | 0.0149 ^{ns} | 0.0132 ^{ns} |
| BY | −0.0761 | −0.1373 | 0.0441 ^{ns} | 0.3085 | 0.1450 | 0.1200 |
| BI ^{\$} | −0.0594 | −0.1039 | 0.0110 ^{ns} | 0.2606 | 0.0134 ^{ns} | 0.2742 |

- Parameter estimates for matrix **A** of the multivariate LogMEM(1,1)₁, fitted to 5-min realised volatility on Coinbase (CB), Binance Spot (BI^S), Huobi (HU), Binance USD-perpetual (BI^T), Bybit (BY) and Binance USD-perpetual (BI^{\$})
- Column denotes emitting exchange, row denotes receiving exchange
- Diagonals in **red** capture flows back into exchange
- Superscript ^{ns} indicates estimate is not significant at 1%
- Data period 1 January to 31 March 2021.

Answers to Research Questions



- Where does bitcoin volatility emerge?
Almost all from Binance Asia, mostly from the tether perpetual
- To which exchanges does volatility transmit?
Bybit and the spot exchanges Coinbase and Binance US
- Do volatility flows change over the course of the day?
Yes, they increase at time of funding payments on perps

4. Bitcoin Options and Variance Swaps

- 90% of bitcoin options traded on [Deribit](#)
- Founded in Netherlands 2016 but moved to Panama in 2021
- Deribit also lists futures and **perpetuals** – and similar ether products
- 1-day, 2-day, 1-week, 2-week, 3-week, monthly and quarterly expires
- Nominal value is 1 bitcoin, settlement price based on XBT index
- Strikes in US dollar but **margin account and cash settlement in bitcoin**

Other bitcoin options centralised exchanges:

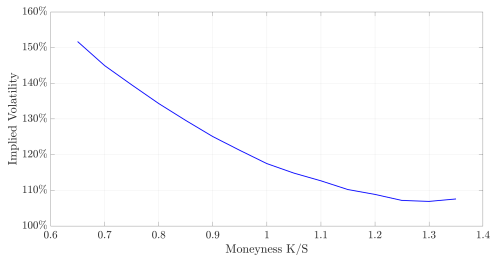
- OKEEx and Bit.com also offer **inverse** options (bitcoin)
- CME* and LedgerX* offer direct options (US dollar)
- Huobi and Binance offer direct options (tether)

* Regulated

Deribit Options: Trading Volume and Implied Volatilities

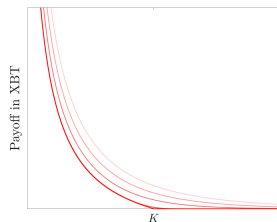
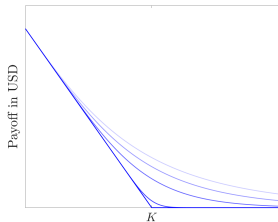
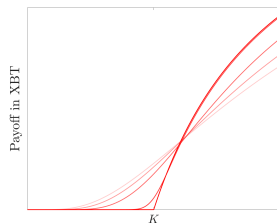
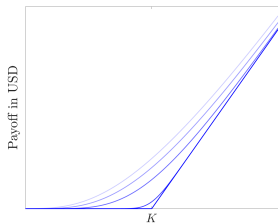
Table: Total Volume and Percentages of Total by Maturities

| Time to Maturity | 2017 | 2018 | Year 2019 | 2020 | 2021 |
|---------------------------|--------|--------|--------------|--------|--------|
| Short Term: 1-7 days | 30.61% | 20.81% | 34.25% | 41.57% | 38.95% |
| Medium Term: 8-21 days | 19.71% | 15.59% | 25.84% | 27.17% | 30.61% |
| Long Term: \geq 22 days | 49.68% | 63.60% | 39.91% | 31.26% | 30.43% |
| Total (\$ billion) | 0.26 | 1.43 | 7.88 | 44.29 | 96.19 |



28-day Maturity Skew on 19 May, 3pm UTC

Pay-Offs to Direct and Inverse Options



Pay-Offs to Direct vs Inverse Options

| $K = 100$ | Bitcoin Price, F | Direct (US dollar) | Inverse (bitcoin) |
|-----------|--------------------|--------------------|-------------------|
| Call | 100 | 0 | 0 |
| | 200 | 100 | 0.5 |
| | 500 | 400 | 0.8 |
| | 1000 | 900 | 0.9 |
| Put | 1 | 99 | 99 |
| | 10 | 90 | 9 |
| | 50 | 50 | 1 |
| | 100 | 0 | 0 |

Inverse call pay-off in bitcoin is **capped at 1**:

$$\frac{1}{F} \max(F - K, 0) = \max\left(1 - \frac{K}{F}, 0\right) = K \max\left(\frac{1}{K} - \frac{1}{F}, 0\right)$$

Inverse put pay-off in bitcoin $\rightarrow \infty$ as $F \rightarrow 0$:

$$\frac{1}{F} \max(K - F, 0) = \max\left(\frac{K}{F} - 1, 0\right) = K \max\left(\frac{1}{F} - \frac{1}{K}, 0\right)$$

Call (put) is put (call) with underlying $\frac{1}{F}$ and strike $\frac{1}{K}$

Optimal Dynamic Delta Hedging

Write option price $f(K, T|F, \sigma) := f$ and implied volatility $\theta(K, T|F, \sigma)$ similarly drop dependence on option strike K , maturity T , underlying F and process volatility σ for delta δ_{BS} and vega ν_{BS}

Adjust delta to account for volatility dynamics:

$$\delta_{adj} = \frac{\partial f}{\partial F} + \frac{\partial f}{\partial \sigma} \frac{\partial \sigma}{\partial F} = \delta_{BS} + \nu_{BS} \sigma_F$$

Derman et al. (1996) **sticky models** set $\sigma_F = \kappa \theta_K$, a regime-dependent multiple of the **slope of the smile** where:

$$\kappa = \begin{cases} 0 & \text{(Sticky Strike – Range Bounded)} \\ 1 & \text{(Sticky Tree – Crash Market)} \\ -K/F & \text{(Sticky Delta – Stable Trending)} \end{cases}$$

The Minimum Variance (MV) delta of Lee (2001) which minimizes the hedging error variance has the **opposite adjustment** to the sticky delta model

Empirical Design in Alexander and Imeraj (2021b)

Short one option and dynamically delta hedge

Parameters:

- Hedging model: SS, ST, SD, MV
- Option moneyness range – 0.7 to 1.3
- Option maturity – 7, 14, 28, 60 days
- Hedging instrument – the Deribit perpetual
- Rebalancing frequency – 8 hrs or less
- Rebalancing costs – Up to 10bps

See Alexander et al. (2021a) for modelling the impact of auto-liquidations on optimal hedging

Data

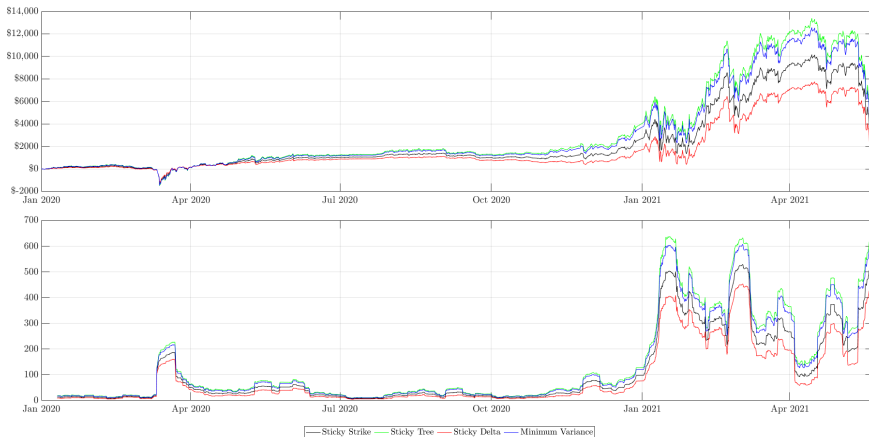
- Mid quote of Deribit futures and options at 1-min frequency
- PCHIP interpolation to synthetic constant-maturity prices
- Perpetuals prices implied through PCP
- PCHIP interpolation of implied volatilities by moneyness
- Slope of smile by derivative of fitted cubic at that point

Preliminary results (P&L in USD)

- Moneyness $K/F = 0.7, 0.75, \dots, 1, 1.25, 1.3$
- Rebalancing delta hedge every 1, 2, ..., 8 hours
- Sample period 1 Oct 2020 to 21 May 2021

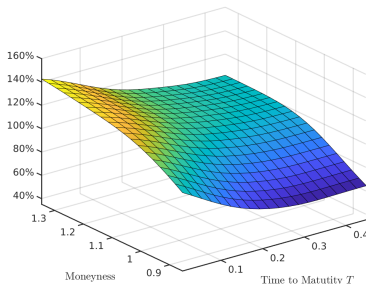
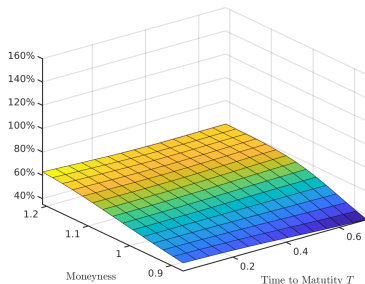
Preliminary Results

OTM puts: $K/F = 0.75$ hedged every 8 hrs



Implied Volatilities

Implied volatilities from out-of-the-money options of different strikes and maturities on 15 April 2019 at 02:00:00 UTC (left) and 16 June at 13:00:00 UTC (right). Surfaces interpolated using shape-preserving splines.



Fair-Value Variance Swap Rate (VIX)

Implied variance from traded options:

$$V_T = T^{-1} \left[2 \sum_i K_i^{-2} \mathcal{Q}(K_i, T) \Delta K_i - \left(\frac{F_T}{K_0} - 1 \right)^2 \right]$$

Interpolation to constant maturity gives fair-value variance swap rate:

$$\text{VXBT}_T = \sqrt{\omega T_1 V_{T_1} + (1 - \omega) T_2 V_{T_2}}$$

with

$$\omega = \frac{n_2 - n}{n_2 - n_1} N$$

$\mathcal{Q}(K_i, T)$ is the price of OTM option with maturity T and strike K_i

F_T is the price of the futures contract with maturity T

K_0 is separation strike, i.e. puts for strikes $K_i < K_0$ and calls for strikes $K_i > K_0$

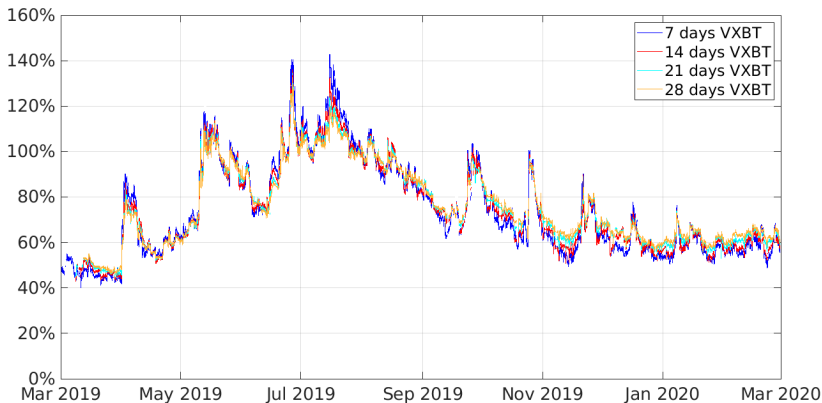
N is the number of seconds in a year

n_i is the number of seconds until maturity T_i , for $i = 1, 2$

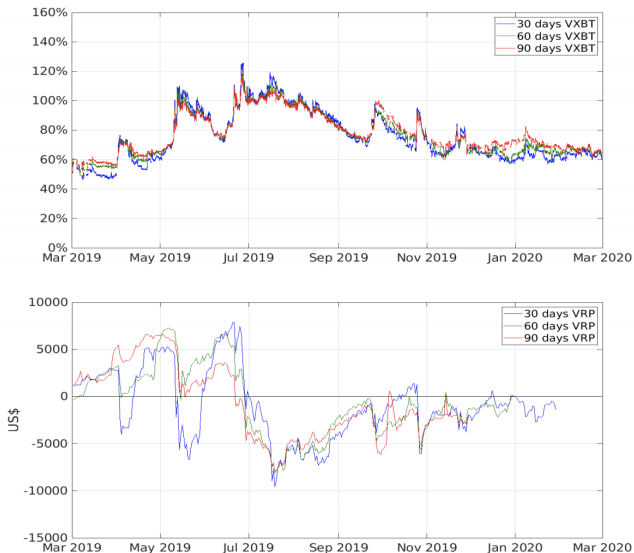
n is the number of seconds until maturity T

Variance Swap Rate Term Structure (Short End)

Alexander and Imeraj (2021a)



Bitcoin Variance Risk Premium per \$1 Notional



CryptoCompare Live-Streamed BVIN Index



The **Bitcoin Volatility Index (BVIN)** is an implied volatility index that also represents the fair value of a bitcoin variance swap.

The index is calculated by CryptoCompare **using options data from Deribit** and has been developed in collaboration with **Carol Alexander** and **Arben Imeraj** at the University of Sussex Business School. It follows the research design of Alexander and Imeraj (2020) and as such it is the first rigorously constructed index that is suitable for use as a settlement price for bitcoin volatility futures.

You can find more information on the **Bitcoin Volatility Index (BVIN)** methodology page.

Looking to create an index? We offer custom and white label cryptocurrency index solutions, developed using CryptoCompare data and administered by MVIS.

BVIN

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